ELEC 201 Videos

Charge/Current/Voltage

Charge

$q_e = -1.602 \times 10^{-19}\,C$

$q_p = 1.602 \times 10^{-19}\,C$

q

- measured in Coulombs (C)

$q = \int_{t_1}^{t_2} i\,dt$

Current

i

- measured in A, C/s

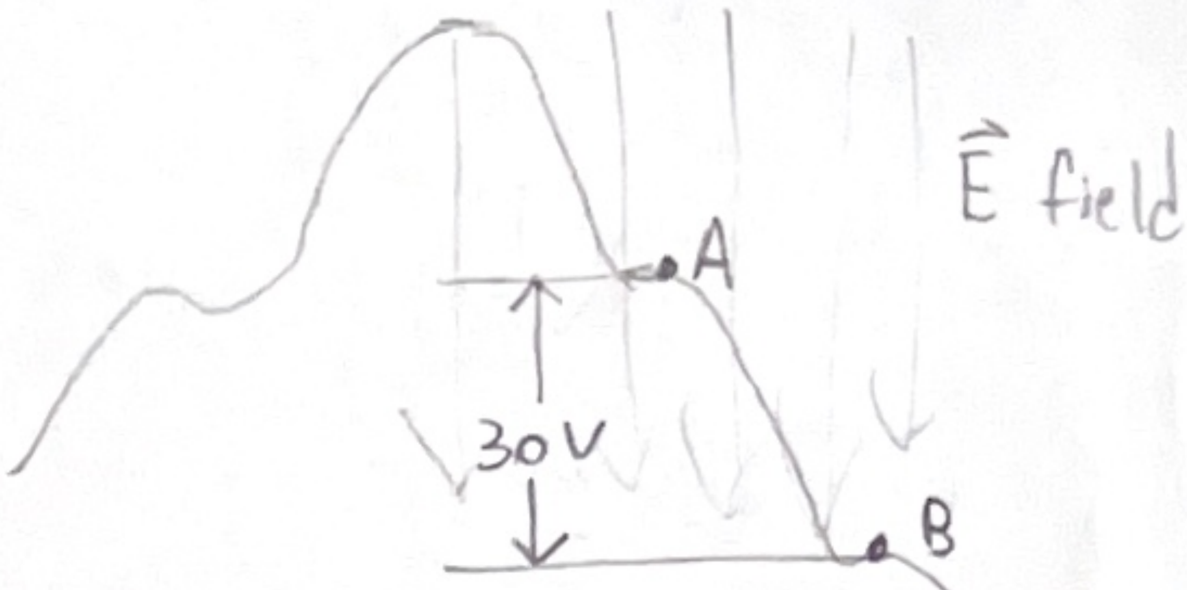
ampere = C/s

Voltage

V

- measured in Volts
- Electric height

Volt = J/C



- if A and B are both 1C of charge
- A→B, delivers 30J of energy
- B→A, costs 30J of energy

Average Current

$q(t)$

- How many net C of charge have passed the checkpoint to the right at a given time

- $i_{avg} = \frac{\Delta q}{\Delta t}$

instantaneous Current

$i(t) = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t}$

$i = \frac{dq}{dt}$

Energy

W

- measured in Joules

$W = \int_{t_1}^{t_2} Vi\,dt$

Power

P

- measured in Watts

Watt = J/s

$P = Vi$

$P = \frac{dW}{dt}$; $t = \frac{W}{Vi}$

$W = \int_{t_1}^{t_2} p\,dt$

Current flowing:

uphill → Delivering power

downhill → absorbing power

Tellegens thereom

$\sum P_{abs} - \sum P_{del} = 0$

- Total absorbed power equals total delivered power in a circuit

$P_{abs} = (V_o - V_d)\,i$

Circuits

Branch

- a link between two proper nodes
- may have zero or more binary nodes

Binary Node

- Does not affect the shape of the circuit

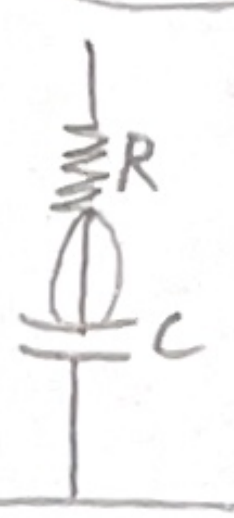
Atomic Branch

- Branch with no binary nodes

Branch

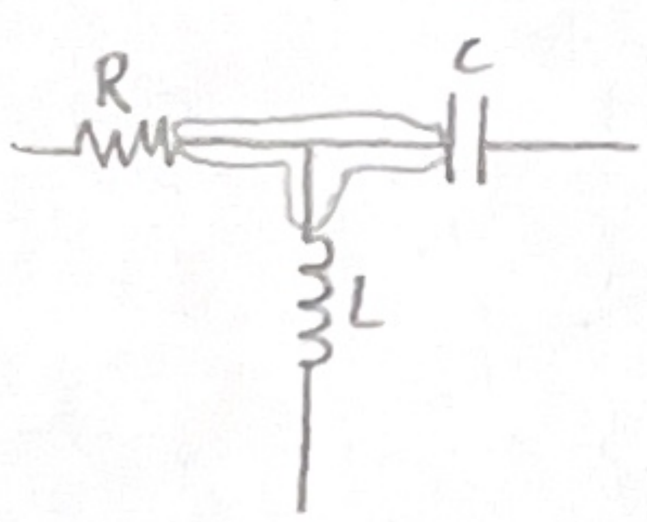
- The path that joins two true nodes. Atomic if no binary nodes inside

Binary Node



(true) Node

- 3 or more



Series

- Elements in the same branch are in series

Parallel

① Both elements are in atomic branches

② Both atomic branches share the same true nodes. Must share both

Series/Parallel

- Two elements in series have the same current.

But two elements that have the same current aren't necessarily in series

- Two elements in parallel have the same voltage but not the other way

Circuit with no true Nodes

- Promote one B-node to rank of true node (arbitrary)

Active Element

Battery, generator, etc, sources

Deliver power (usually)

Passive Element

Resistor, inductor, capacitor

Absorb power (sometimes)

Resistance

- resists currents i

Inductance

- resists changes in currents $\frac{di}{dt}$

Capacitance

- Resists changes in Voltages $\frac{dV}{dt}$

Passive element

- Assume water falls from the sky

Ohm's Law
- R in ohms (Ω)

$\Omega = V/A$

① Resistance
② Power rating

Inductor
- L in Henrys (H)

$V = L\frac{di}{dt}$

① Inductance
② Current rating

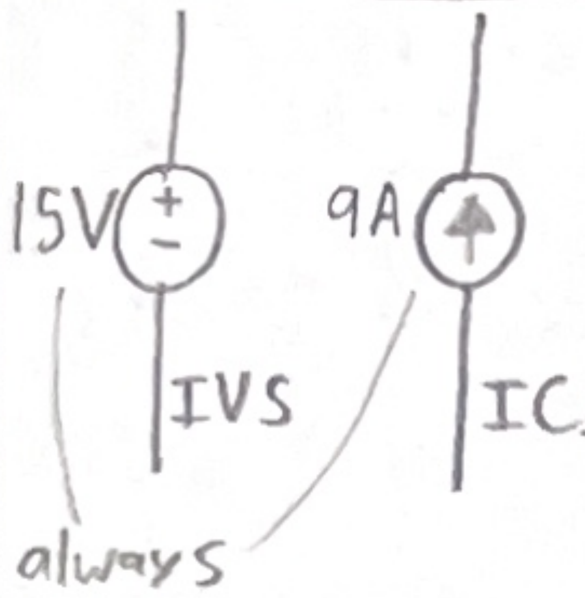
Capacitor
- C in Farads (F)

$i = C\frac{dv}{dt}$; $V = \frac{q}{C}$

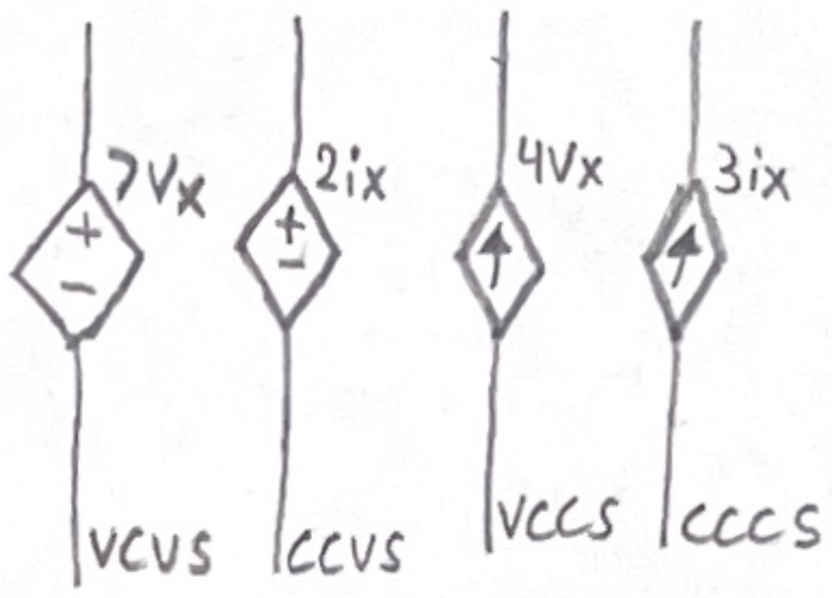
① Capacitance
② Voltage rating

Active (Sources)

Independent:



Dependent:



Kirchoff's Current Law

$\sum_{node} i_{in} = \sum_{node} i_{out}$ $\sum_{Gauss} i_{in} = \sum_{Gauss} i_{out}$

$\sum_{Bndry} i_{in} = \sum_{Bndry} i_{out}$

Super Node
- gaussian surface that may cover many nodes

Inverses

Conductance:

$G = \frac{1}{R}$ - measured in Siemens

Inductance^{-1}:

$\Gamma = \frac{1}{L}$ - measured in H^{-1}

gamma

Capacitance^{-1}:

$T = \frac{1}{C}$ - measured in F^{-1}

$\sum_{loop} V_{up} = \sum_{loop} V_{down}$

Kirchoff's Voltage Law
- a loop is a closed path that only crosses a node once, ie: arrive in it and leave out of it

Window pane (Mesh)
- a loop that does not contain any loops inside

Addition Simplification

$R_{eq} = R_1 + R_2 ... + R_n$ SERIES
$L_{eq} = L_1 + L_2 ... + L_n$ SERIES
$V_{source} = V_1 + V_2 ... + V_n$ SERIES
$C_{eq} = C_1 + C_2 ... + C_n$ PARALLEL
$i_{source} = i_1 + i_2 ... + i_n$ PARALLEL

Inverse Simplification

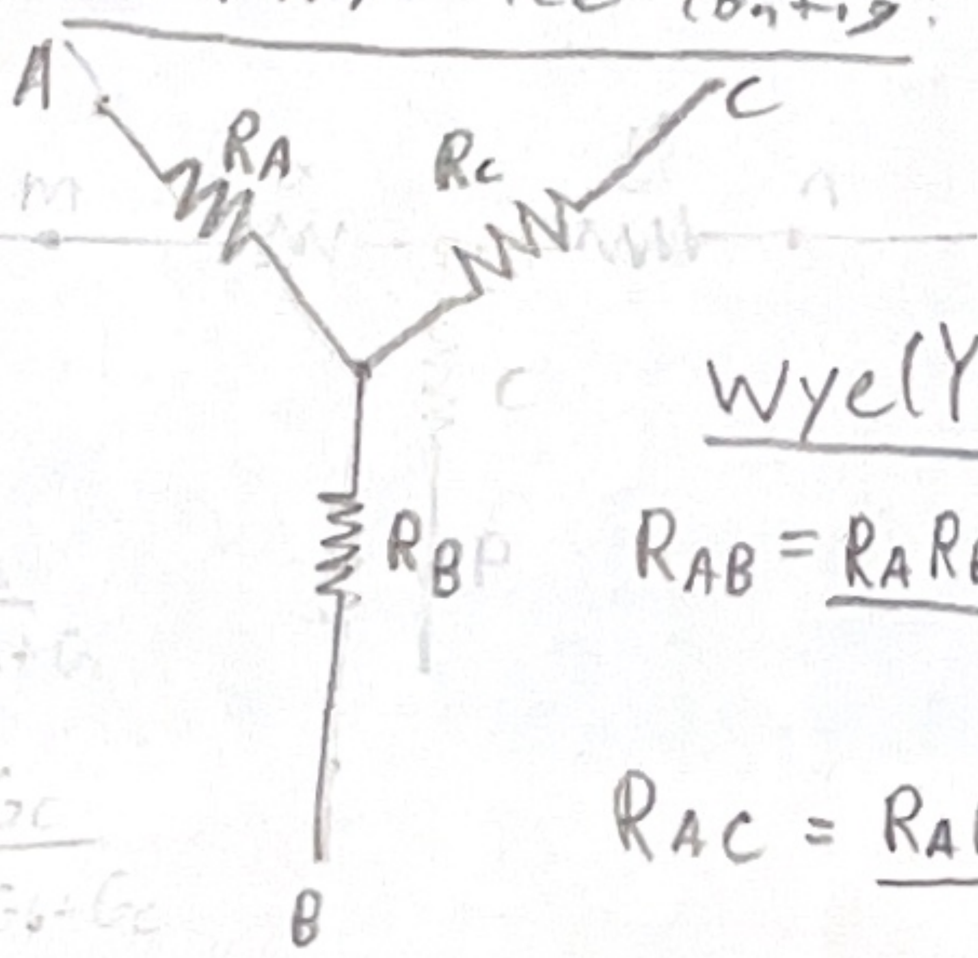
$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} ... + \frac{1}{R_n}\right)^{-1}$ PARALLEL
$L_{eq} = \left(\frac{1}{L_1} + \frac{1}{L_2} ... + \frac{1}{L_n}\right)^{-1}$ PARALLEL
$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} .... + \frac{1}{C_n}\right)^{-1}$ SERIES

*midterm demonstrate not on slides

Neither in Series/Parallel

Triangle/Delta/Pi Config:

Star/Wye/tee config:



Δ Delta → Wye (Y):

$R_A = \frac{R_{AB}R_{AC}}{R_{AB}+R_{AC}+R_{BC}}$

$R_B = \frac{R_{AB}R_{BC}}{R_{AB}+R_{AC}+R_{BC}}$

$R_C = \frac{R_{AC}R_{BC}}{R_{AB}+R_{AC}+R_{BC}}$

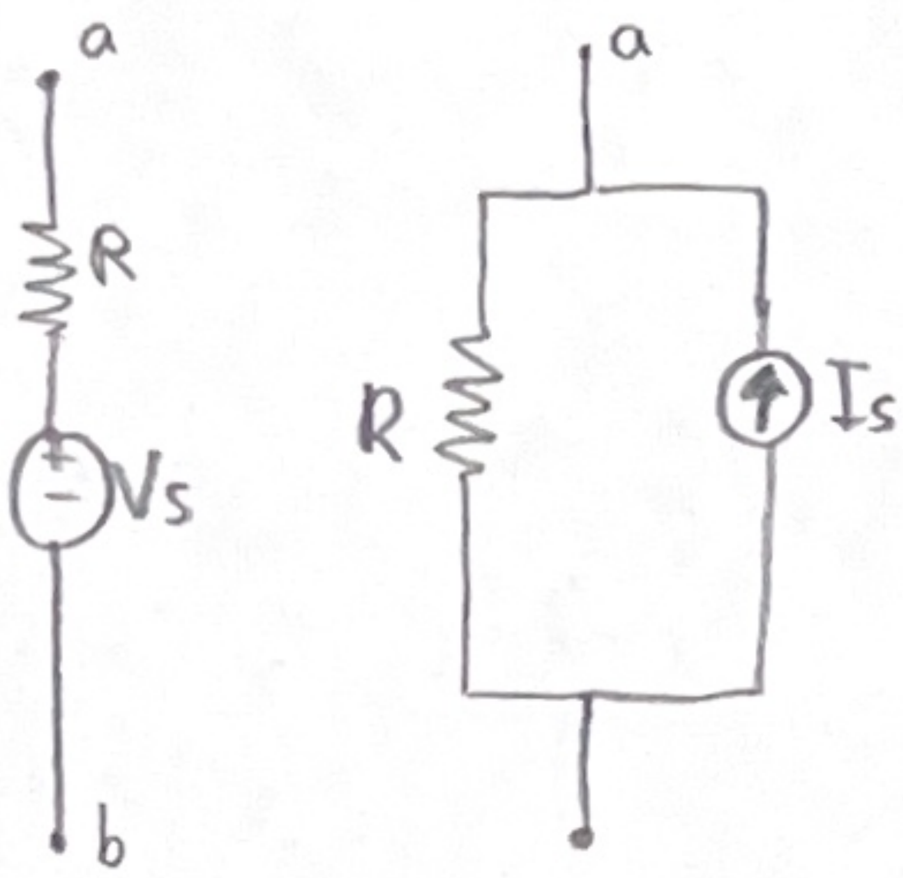
Wye (Y) → Δ Delta:

$R_{AB} = \frac{R_AR_B + R_AR_C + R_BR_C}{R_C}$

$R_{AC} = \frac{R_AR_B + R_AR_C + R_BR_C}{R_B}$

$R_{BC} = \frac{R_AR_B + R_AR_C + R_BR_C}{R_A}$

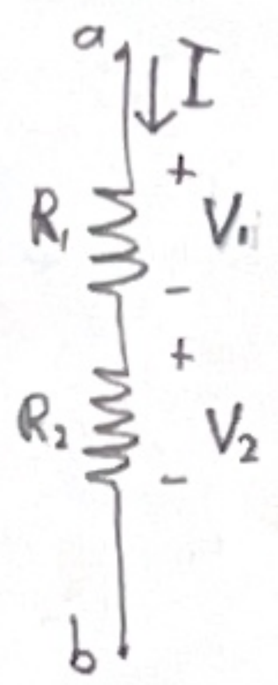
Source Transformations



$V_S = R I_S$

$I_S = \frac{V_S}{R}$

Voltage Divider



$V_1 = R_1 I$

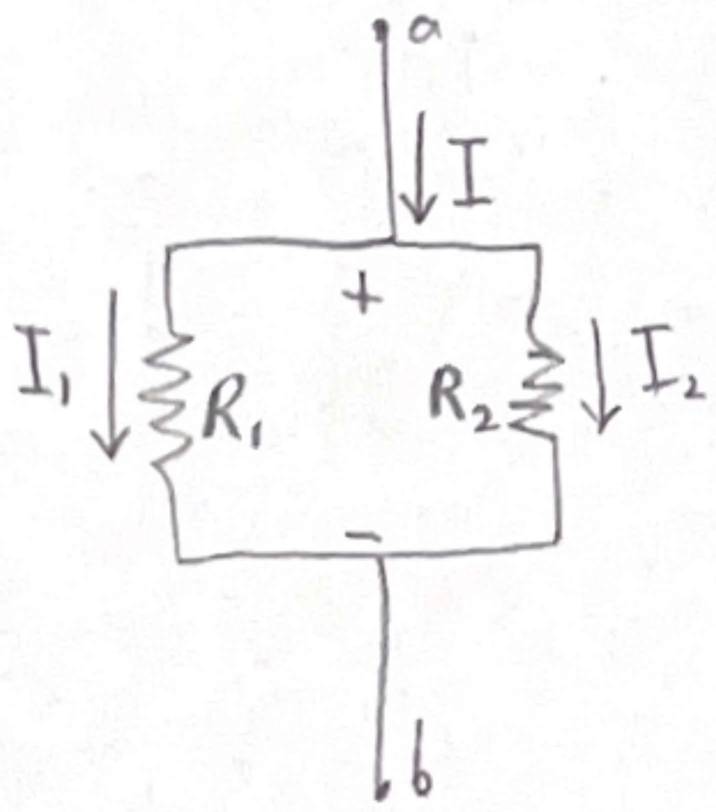
$V_2 = R_2 I$

$V_{ab} = V_2 + V_1$

$V_{ab} = I(R_2 + R_1)$

$\frac{V_1}{V_{ab}} = \frac{R_1}{R_{eq}}$ "Same %"

Current Divider



$I_1 = \frac{V_{ab}}{R_1}$

$I_2 = \frac{V_{ab}}{R_2}$

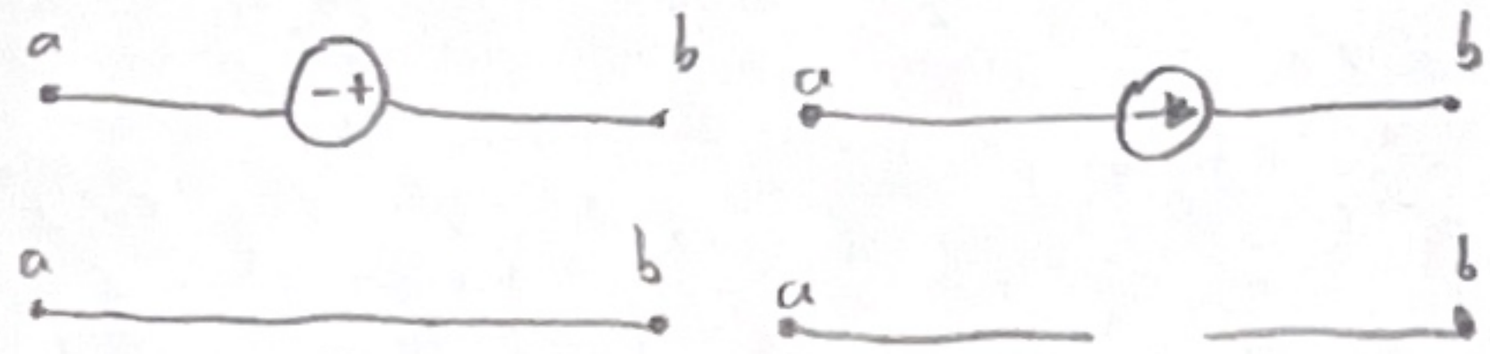
$\frac{I_1}{I} = \frac{R_2}{R_1 + R_2}$ "Same %"

$\frac{I_1}{I} = \frac{G_1}{G_1 + G_2}$

$I = \frac{V_{ab}}{R_1} + \frac{V_{ab}}{R_2} \left(\frac{1}{R_1} + \frac{1}{R_2}\right) V_{ab}$

Superposition

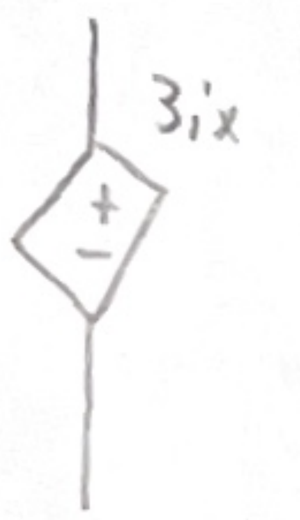
killing a Vsource: killing a Isource:



Short Open

Modified Nodal Analysis

Controlling Variable:

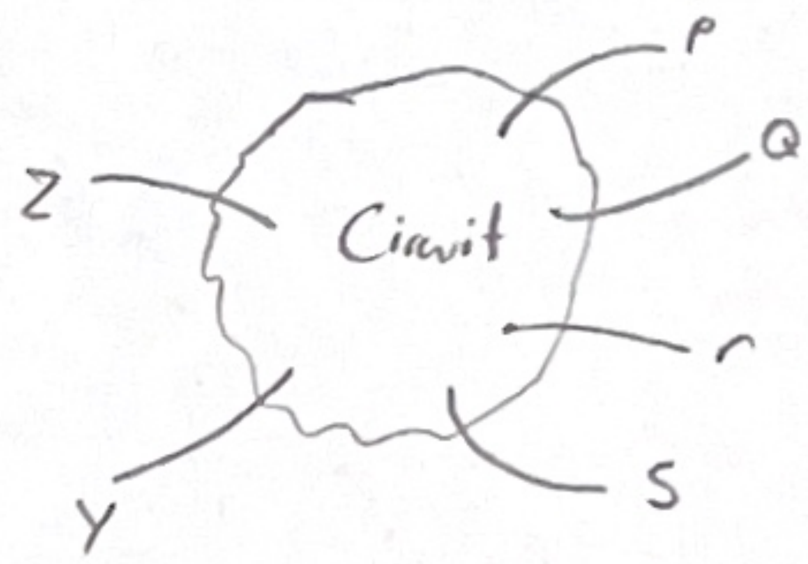


- 3ix is a controlling variable

Evil currents:

- currents in evil branches

Thenevin and Norton Equivilants



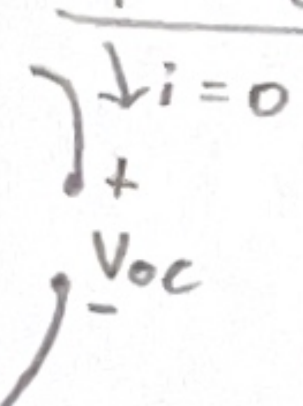
Two nodes = one port (Could be binary or true)

Solution Method

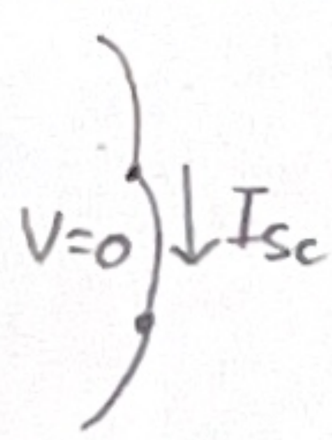
1. Choose REF node
2. Choose R/RV current directions
3. Label every true node (KCL)
4. Label every evil current (EVL)
5. Label every controlling variable (CTL)

(4) New ideal Circuit Elements

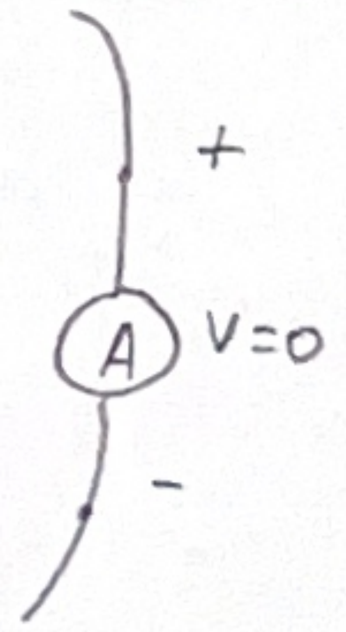
Open circuit:



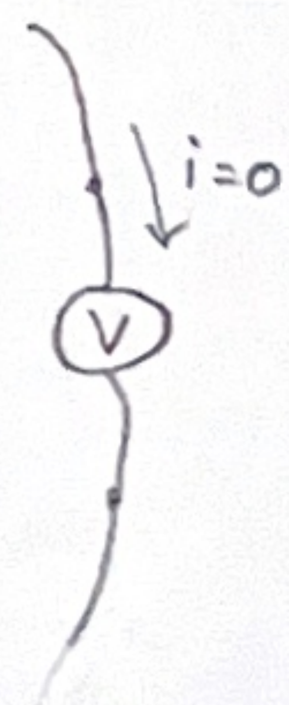
Short circuit:



Ammeter:

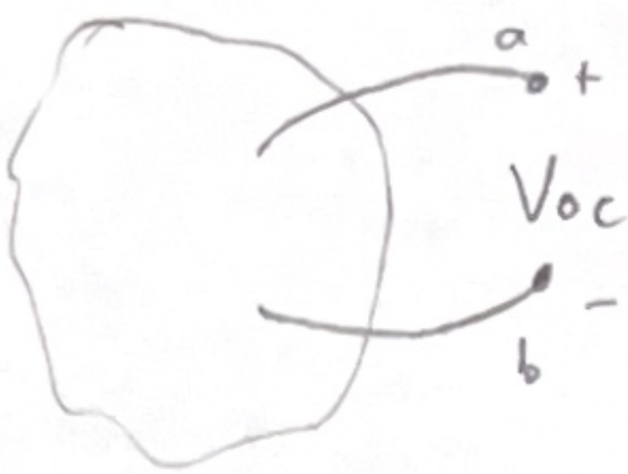


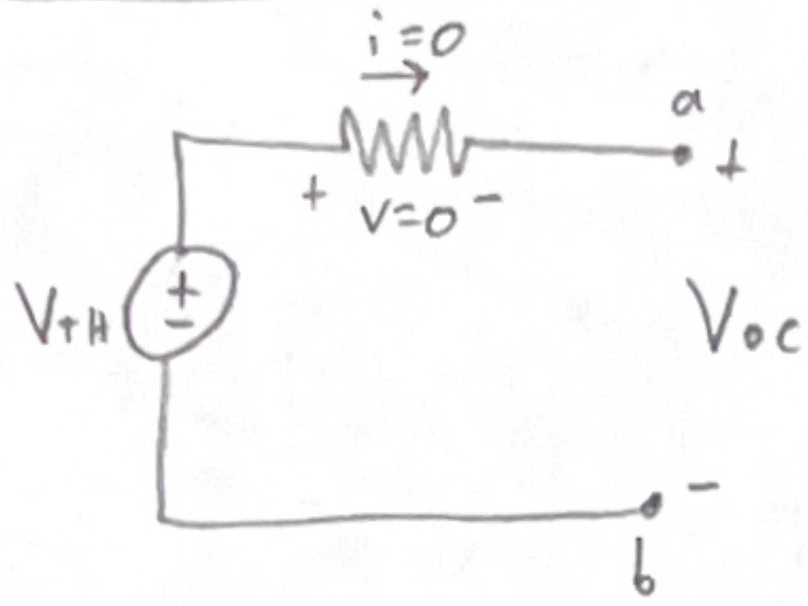
Voltmeter:



Classic Method

Open Circuit test:





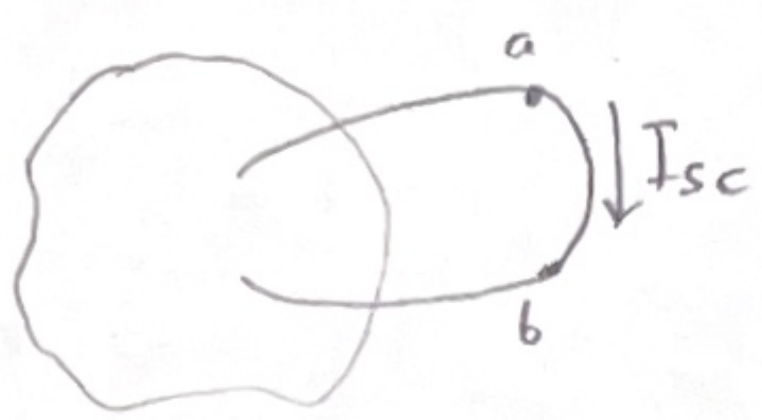
$V_{TH} = V_{oc}$

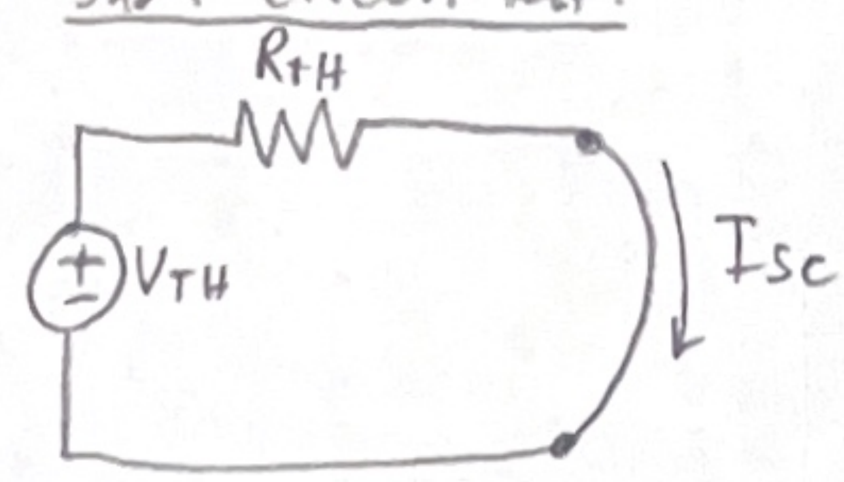
$I_{sc} = \frac{V_{TH}}{R_{TH}}$

$R_{TH} = \frac{V_{TH}}{I_{sc}}$

R_{TH}

Short circuit test:



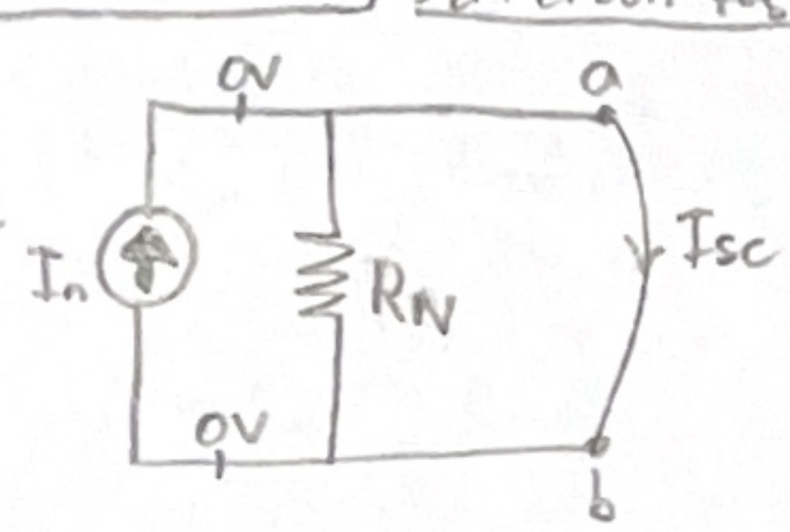


Kill Sources (All independent)

- Find $R_{eq} = R_{TH}$

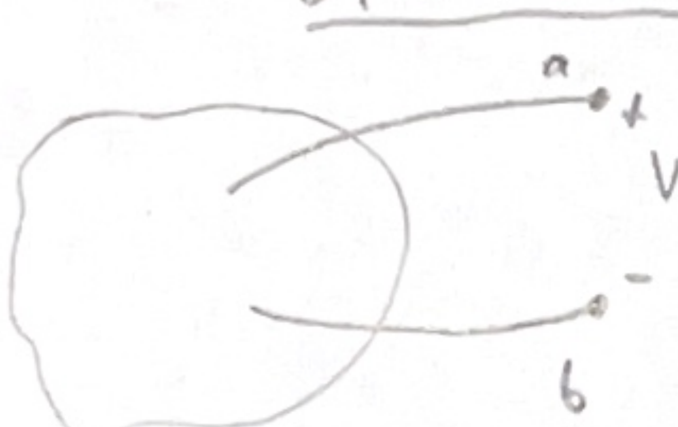
Norton Mayer Equivilant

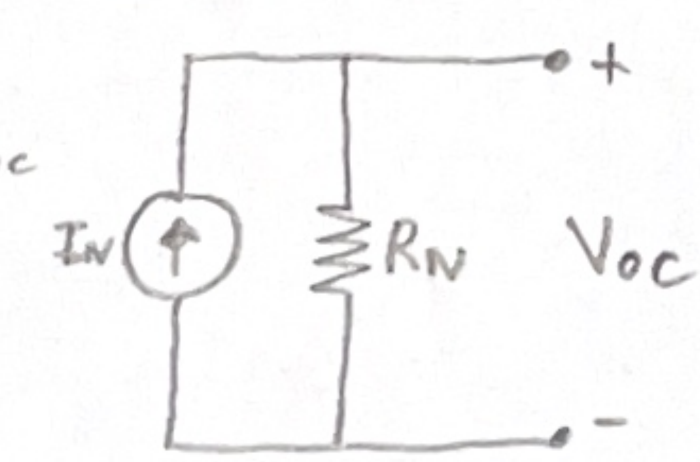
Short circuit test:



$I_N = I_{sc}$

Open circuit test:





$V_{oc} = R_N I_N$

$R_N = \frac{V_{oc}}{I_n}$

$R_n = \frac{V_{oc}}{I_{sc}}$

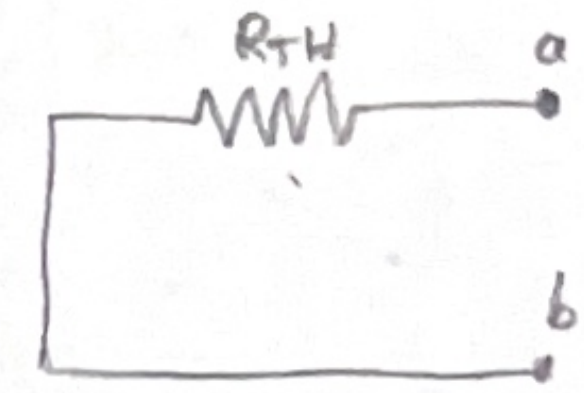
Max Power Criterion

$P_{MAX} = \frac{V_{th}^2}{4R_{th}}$

$P_{MAX} = \frac{I_{sc}^2 R_{th}}{4}$

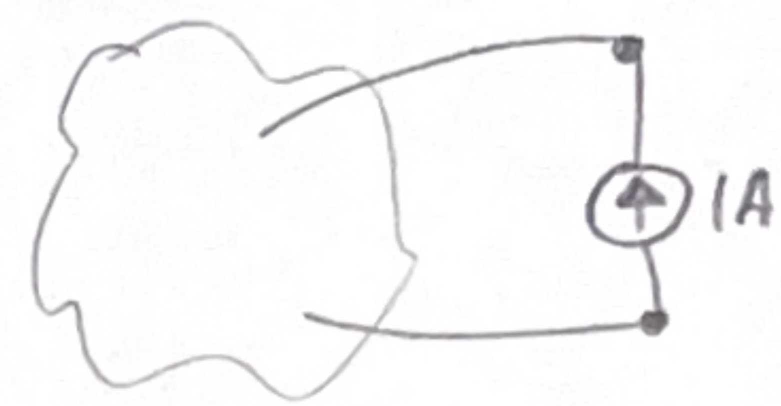
Only Controlled Sources

$V_{oc} = 0$



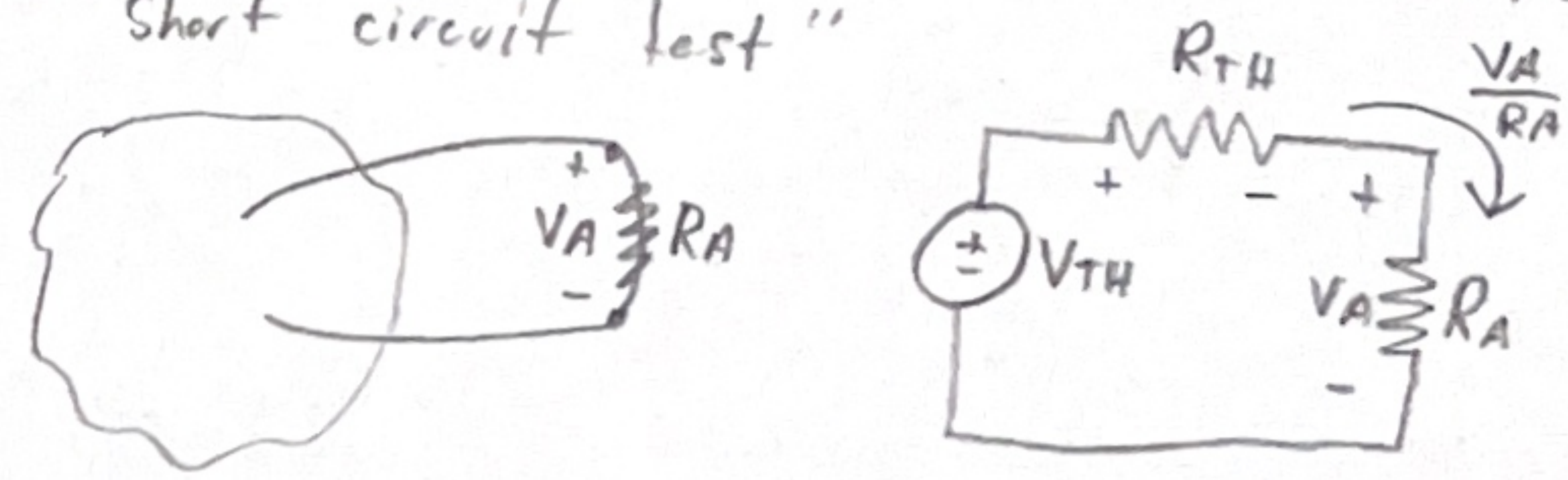
How.... :

apply a 1Amp current between the nodes, solve for R_{TH}



Two Resistors Method

"In real life its dangerous to apply Short circuit test"



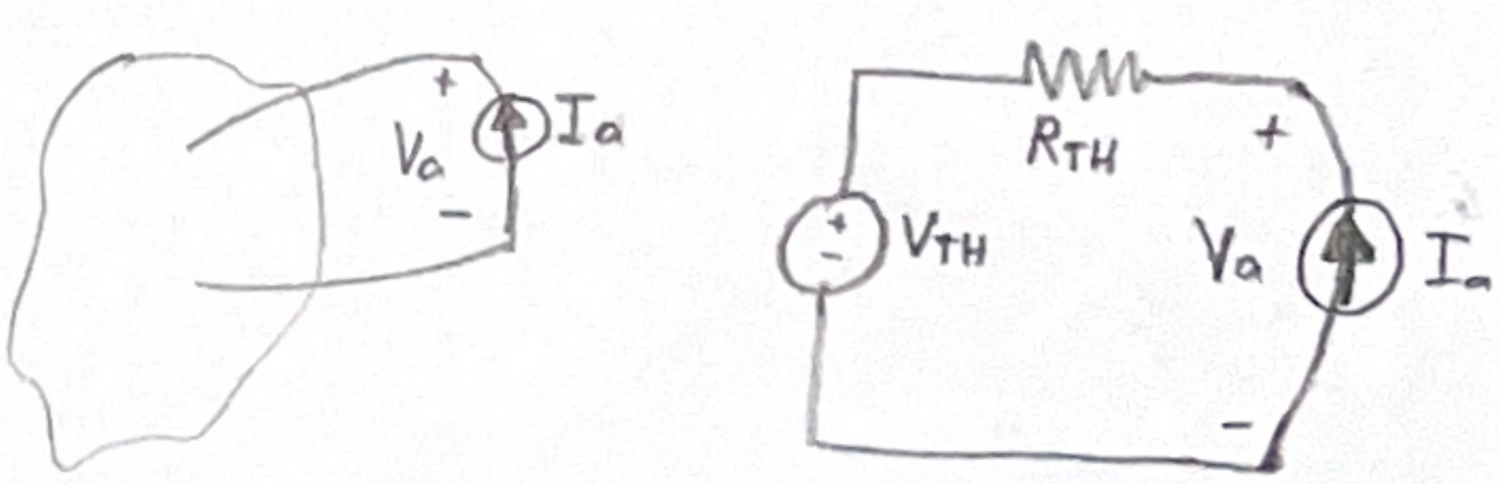
"Do this again with a second Resistor R_B"

SOLVE for V_{TH}, R_{TH}

$V_{TH} - R_{TH}\frac{V_A}{R_A} = V_A$

$V_{TH} - R_{TH}\frac{V_B}{R_B} = V_B$

1A/2A (UBC) Method



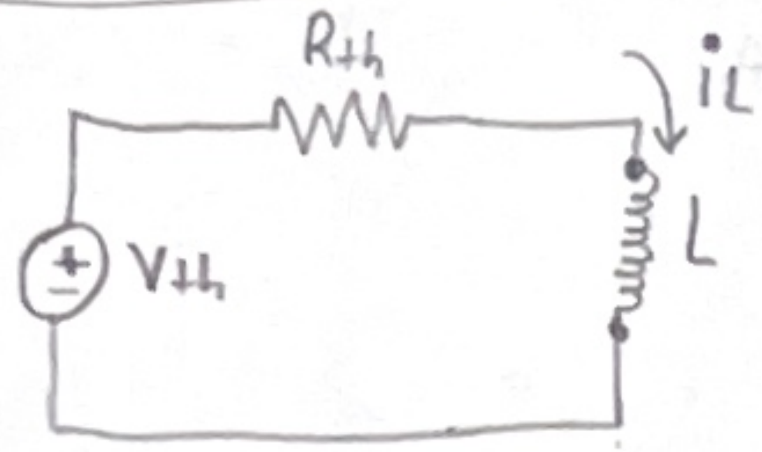
"Repeat with new Isource"

$V_{th} + R_{th} I_a - V_a = 0$

$V_{th} + R_{th} I_b - V_b = 0$

First order Circuits

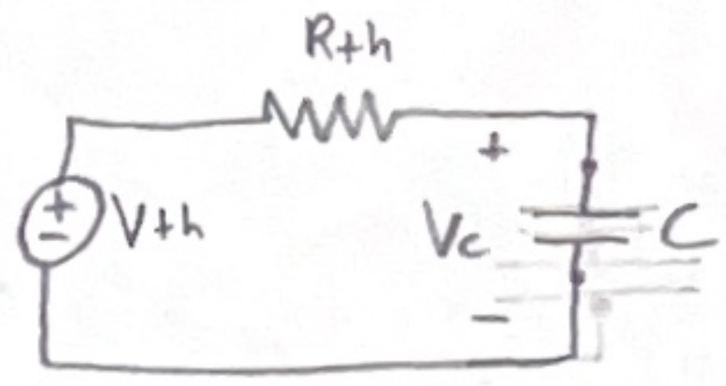
RL Circuit



DE: $L\frac{di_L}{dt} + Ri_L = V_s = 0$

Sol'n: $i_L(t) = (I_o - \frac{V_s}{R})e^{-t/(\frac{L}{R})} + \frac{V_s}{R}$

RC Circuit



DE: $RC\frac{dV_c}{dt} + V_c = V_{th}$

Sol'n: $V_c(t) = (V_{c_o} - V_{th})e^{-t/RC} + V_{th}$

Impedance

$Z_L = Lp$ $\quad$ $*\; p = \frac{d}{dt}$ (s^{-1})

$Z_C = \frac{1}{Cp}$

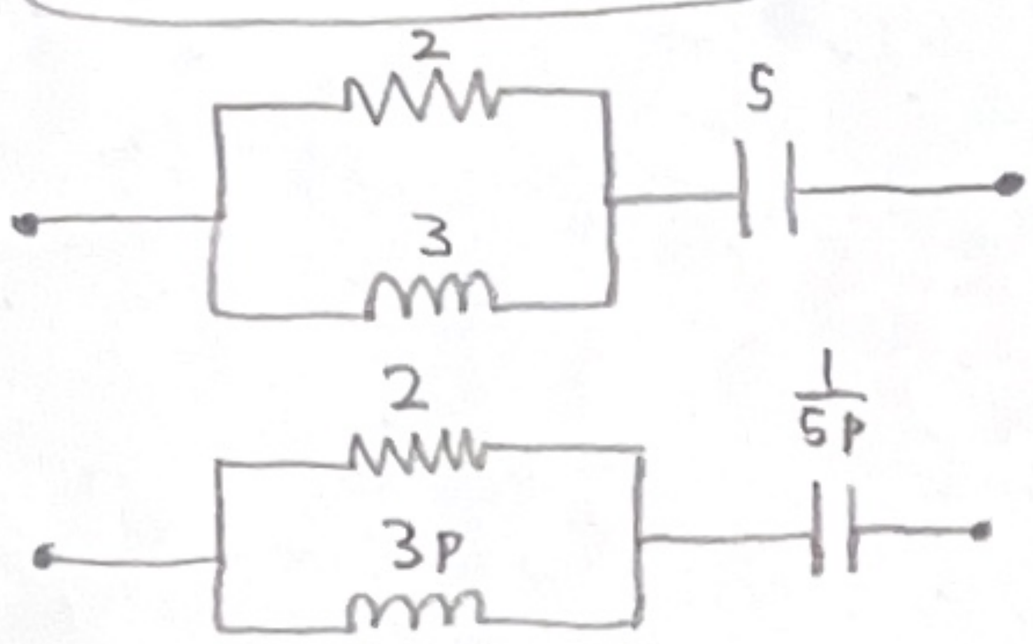
$Z_r = R$

General Ohm's Law:

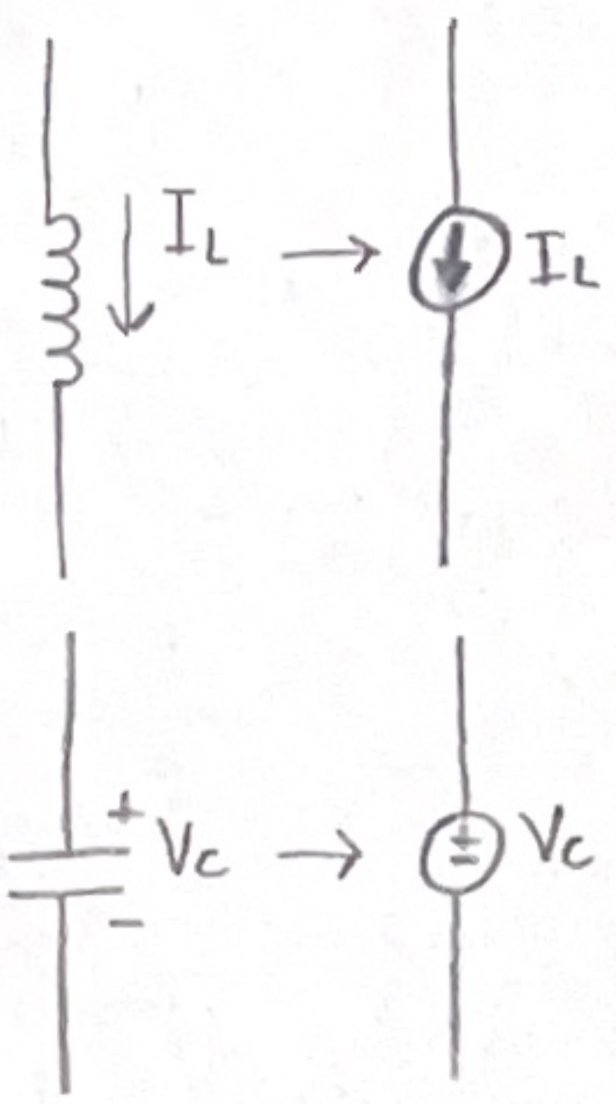
$V = Z \cdot i$

Volts, impedance, current

R/L/C Impedance



Right after we move a switch $t = 0^+$



Capacitor

where V_o is initial voltage

$V_C = V_o e^{-t/RC}$

$Q = CV_o e^{-t/RC}$

$I = \frac{V_o}{R_{th}} e^{-t/RC}$

Inductor

where V_b is final Voltage

$V_L = V_b e^{-tR/L}$

$I = \frac{V_b}{R}(1 - e^{-tR/L})$

$V_c(t) = (V_{co} - V_{cf})e^{-t/RC} + V_{cf}$

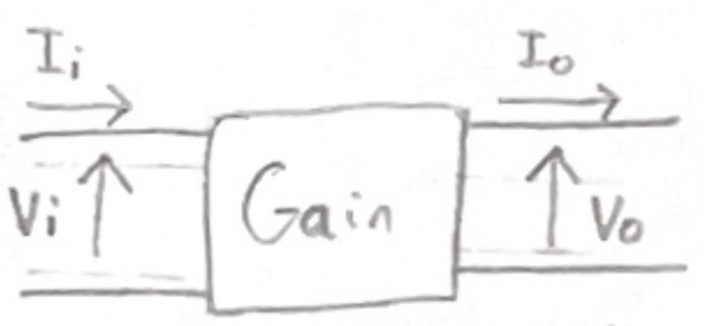
$I_L(t) = (I_{Lo} - I_{Lf})e^{-tR/L} + I_{Lf}$

where V_{co} and I_{Lo} are DCSS of both

$V_{cf} = V_{TH}$ seen by capacitor

$I_{Lf} = I_N$ seen by inductor

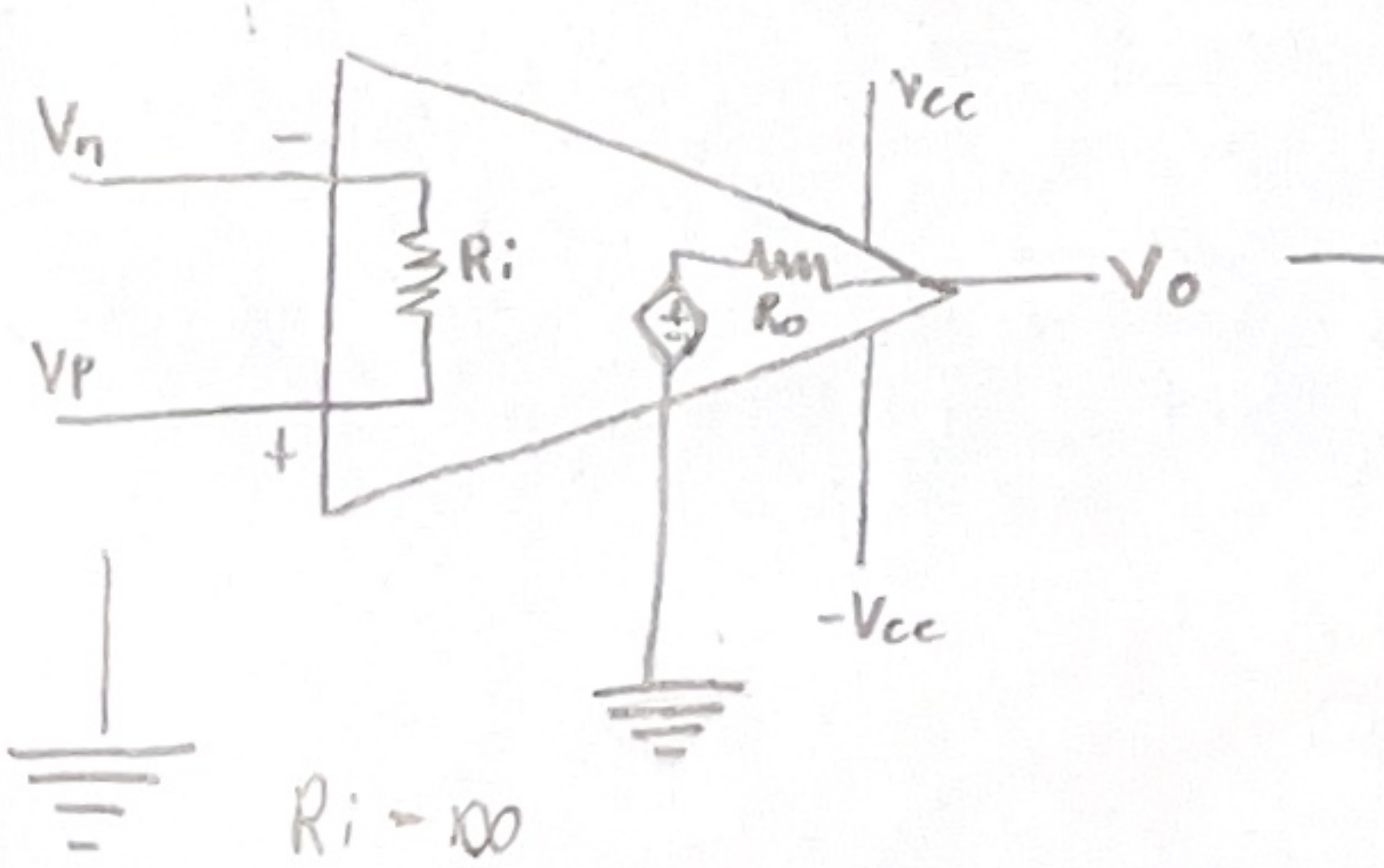
Operational Amplifiers



gains

$A_v = \frac{V_o}{V_i}$ (V/V) $\quad A_I = \frac{I_o}{I_i}$ (A/A) $\quad A_p = \frac{P_o}{P_i}$ (W/W)

$A_v^{DB} = 20\log_{10}\left|\frac{V_o}{V_i}\right| \quad A_I^{DB} = 20\log_{10}\left|\frac{I_o}{I_i}\right| \quad A_p^{DB} = 10\log_{10}\left|\frac{P_o}{P_i}\right|$



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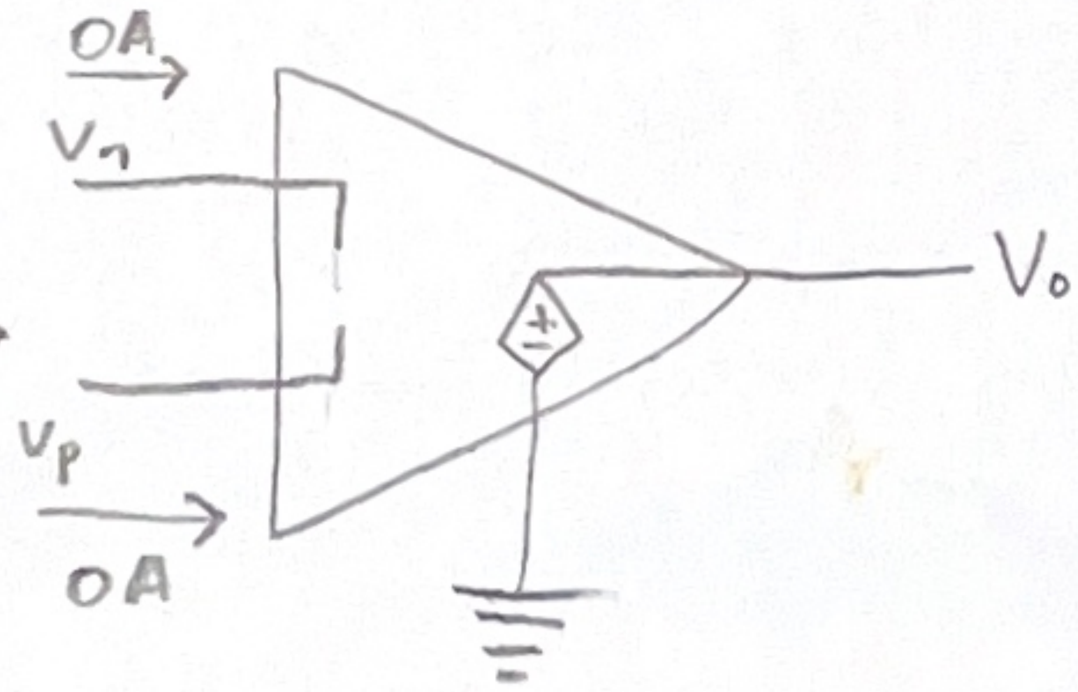


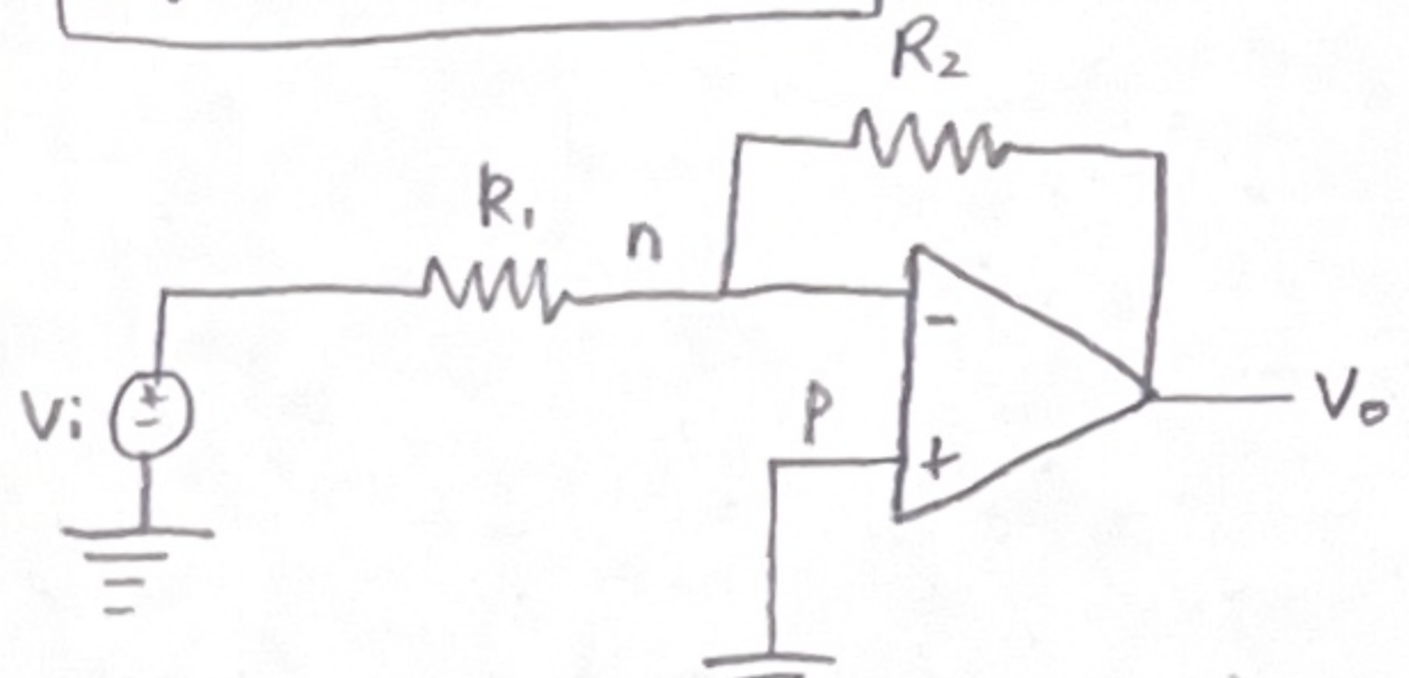
$R_i = \infty$

$R_o = 0$

$A = \infty$ (voltage gain)

$V_o = A(V_p - V_n)$

Negative Feedback



$V_p = V_n$

- as long as no saturation

- connecting the output to the inverting input by passive elements

Inverting amplifier

$V_p = V_n$

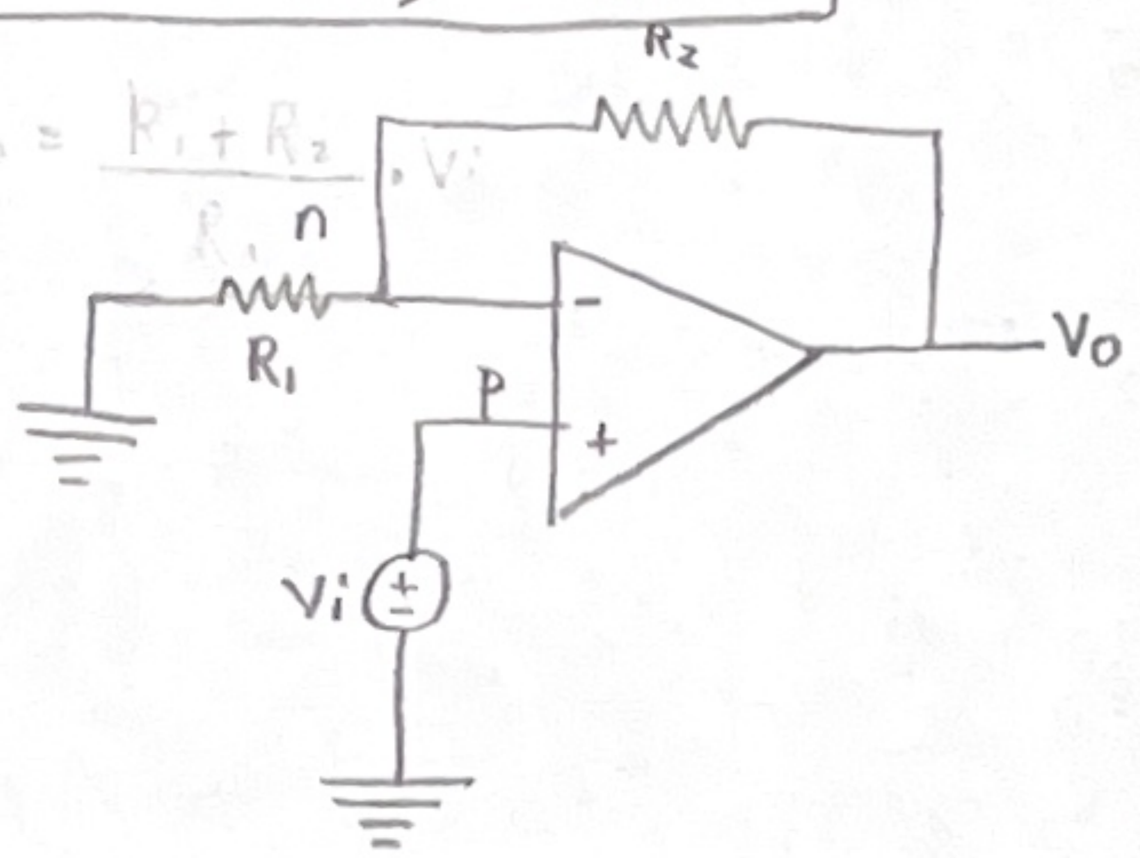
$V_p = 0$

- Don't write a kcl for V_o**

$$n: \frac{V_i - V_n}{R1} = 0 + \frac{V_n - V_o}{R_2}$$

$$\therefore V_o = -\frac{R_2}{R_1} V_i$$

Non-Inverting Amplifier



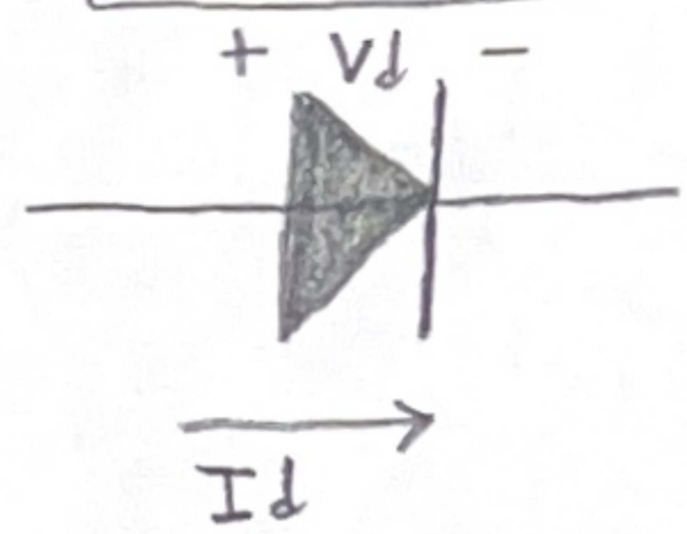
$V_p = V_n$

$V_p = V_i$

$$n: \frac{V_o - V_n}{R_2} = 0 + \frac{V_n}{R_1}$$

$$\therefore V_o = \frac{R_1 + R_2}{R_1} \cdot V_i$$

Ideal Diode



$$I_d = I_o e^{V_D/V_T}$$ constant

Saturation

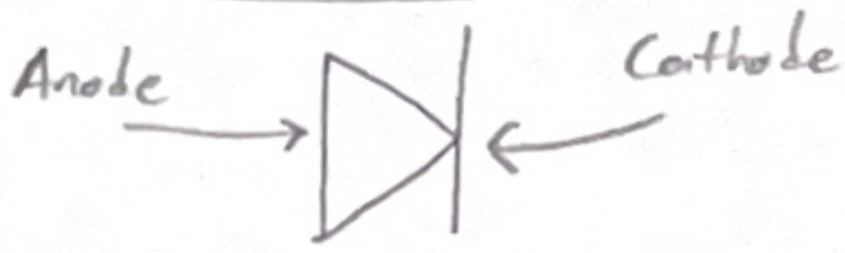
$-V_{cc} \le V_o \le +V_{cc}$

Positive Saturated $\rightarrow V_o = +V_{cc}$, $V_p > V_n$

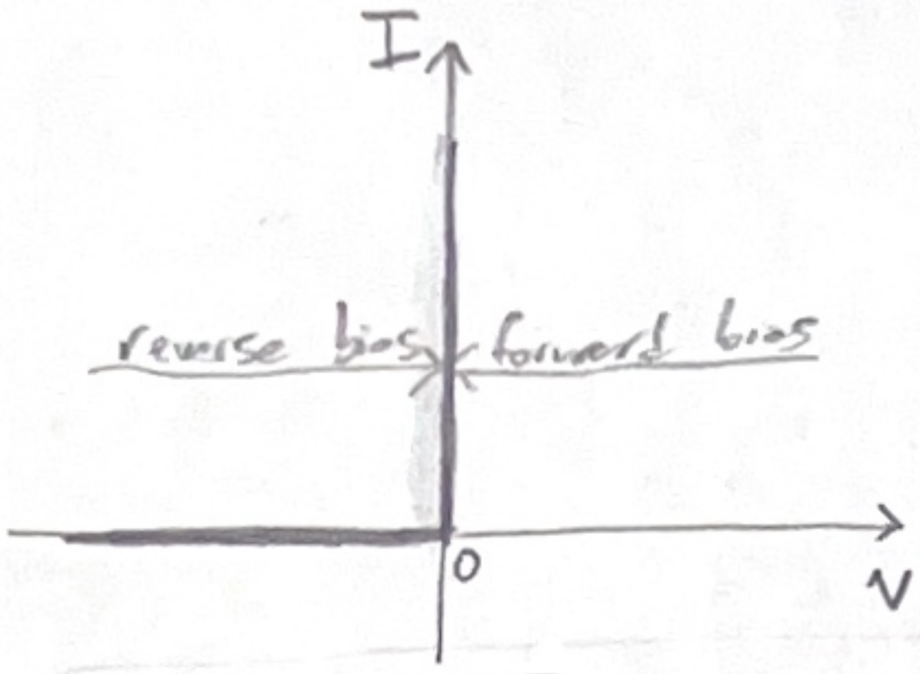
Negative Saturated $\rightarrow V_o = -V_{cc}$, $V_p < V_n$

Diodes

Ideal diode

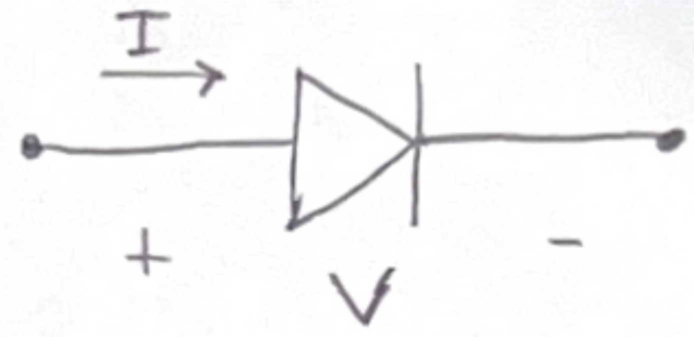


- Allows current to flow in one direction
- No voltage drop across terminals when allowing current to flow



Real diode

- Allows a small amount of reverse current
- Has a small voltage drop when current flowing



(2) modes of operation:

① Reverse bias, $V < 0$, $I = 0$, behaves like open circuit

② Forward bias: $V = 0$, $I > 0$, behaves like short circuit

$$i = I_S\left(e^{\frac{V}{nV_T}} - 1\right)$$

where: $\boxed{i}$ = current through the diode

$\boxed{V}$ = voltage across the diode

$\boxed{I_S}$ = reverse saturation current, very small usually $10^{-12}A$ or $10^{-15}A$

$\boxed{n}$ = empirical constant

$\boxed{V_T}$ = Thermal voltage

$V_T = \frac{KT}{q}$ where: $\boxed{K}$ = Boltzmann's constant

$\boxed{T}$ = absolute temperature

$\boxed{q}$ = base charge

Simplified eqn:

- Because usually $i >> I_S$, $V >> 10nV_T$

$$i \approx I_S e^{\frac{V}{nV_T}}$$

Models

① Ideal Model
② Exponential Model
③ Constant Voltage Drop
④ Piecewise linear
⑤ Small signal Model

Ideal Model

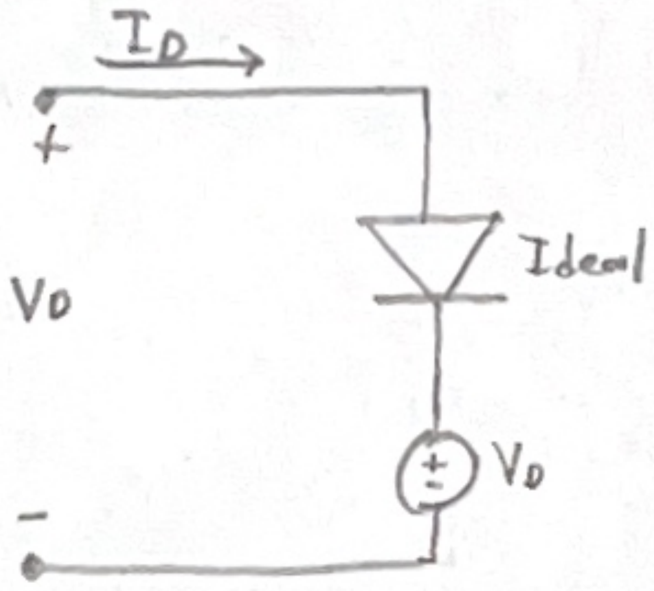
- Try all possibilities
2^n possibilities where n is the number of diodes

Equation Model

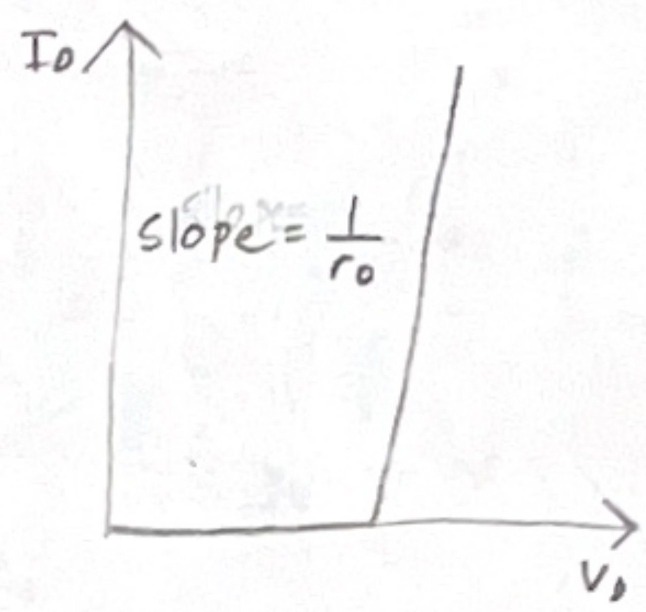
- Make a system of equations
- Most accurate
- Hard without a calculator
- Also called the exponential model

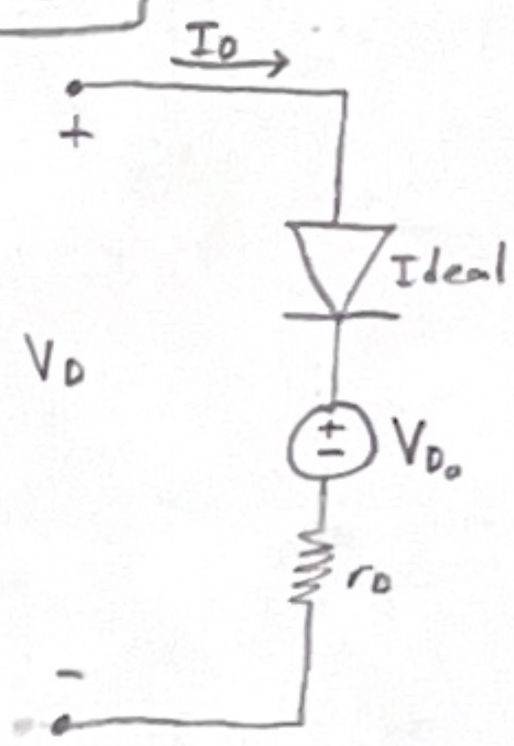
Constant Voltage Drop

- The diode can be replaced with an ideal diode and voltage source in series



Piecewise Linear Model





$$\frac{1}{r_D} = \frac{1mA - 0mA}{0.7V - V_{D0}}$$

- To find V_{D0}

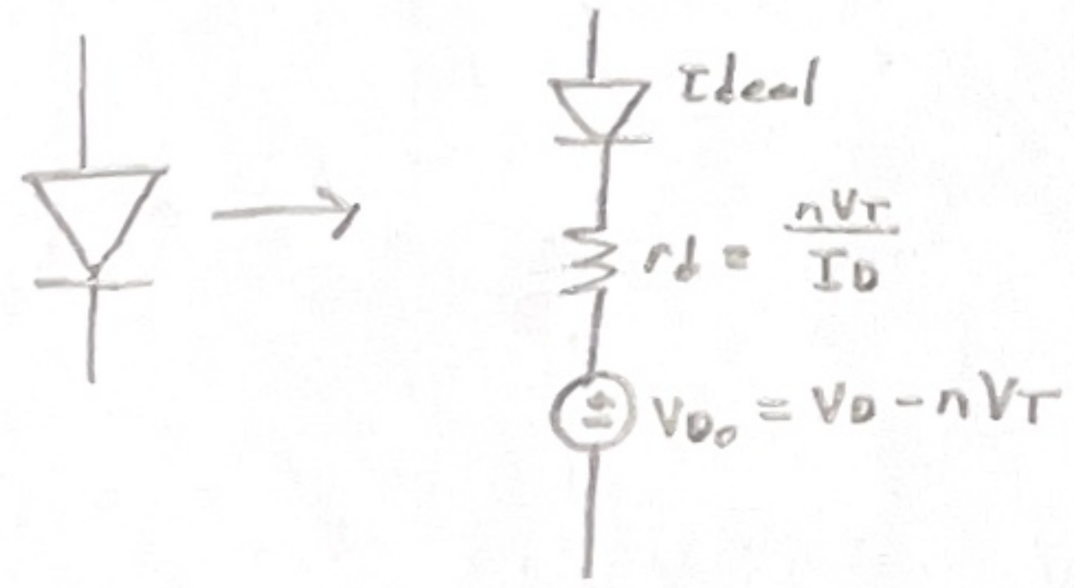
Small Signal Model

① Solve the operating point of the diode in the DC circuit.

To get DC:

- Short all AC voltage sources
- Open all AC current sources
- Short all inductors
- Open all capacitors

② Replace diode with its small signal equivalent in AC circuit

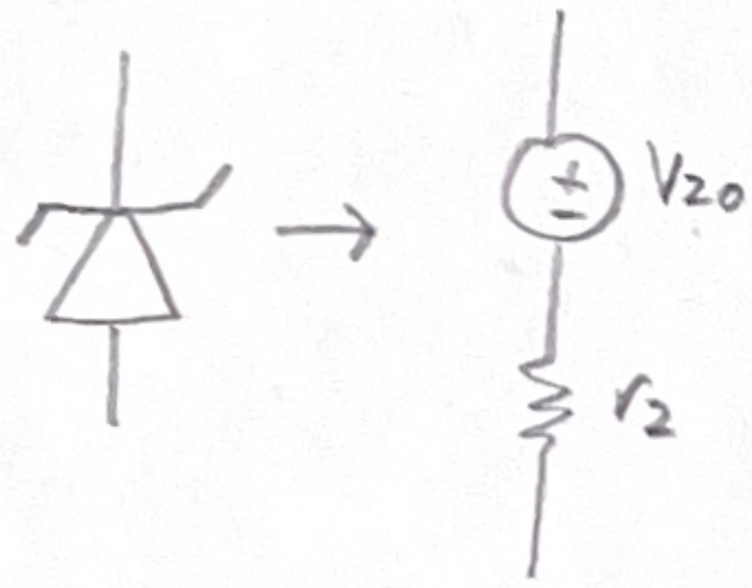


To get AC:

- Short all DC voltage sources
- Open all DC current sources
- Open all inductors
- Short all capacitors

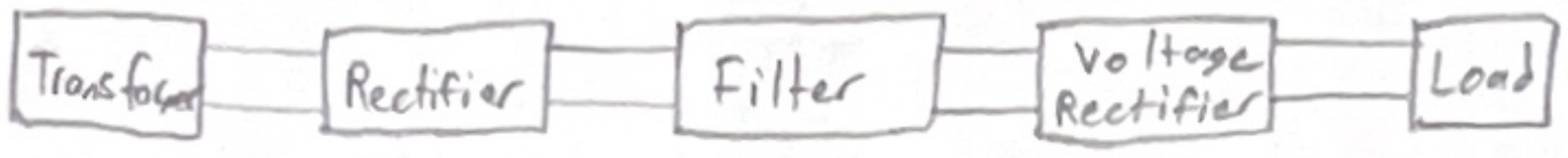
Zener diode

- allows current to flow in reverse at certain voltage



$$V_{Z0} = V_Z - r_Z I_Z$$

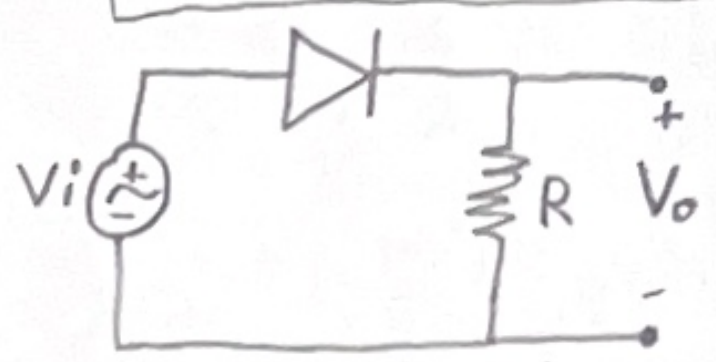
Rectifier Circuits



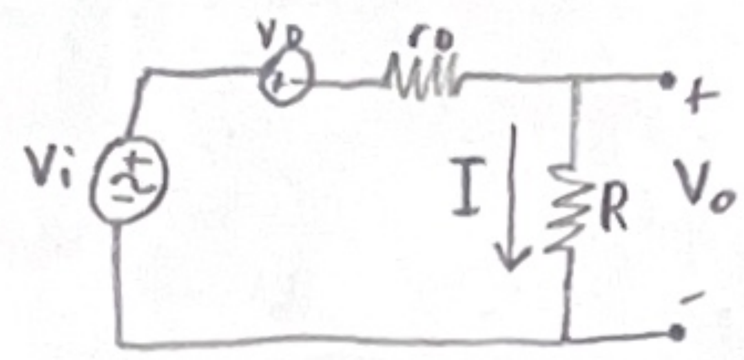
Types of Rectifiers

1. Half-Wave
2. Full-Wave:
   - Center tap
   - Bridge type

Half-Wave Rectifier



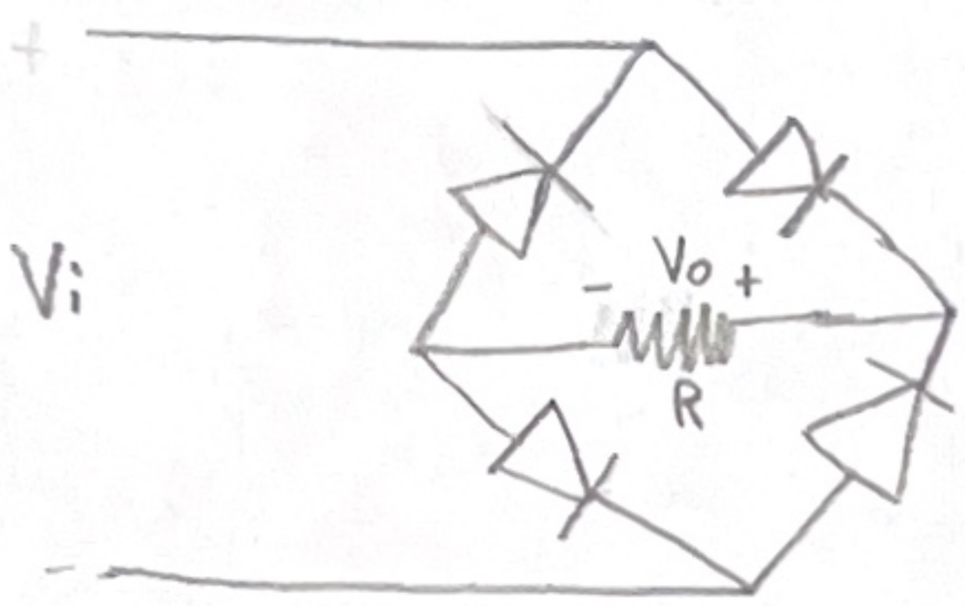
↓ Equivilant



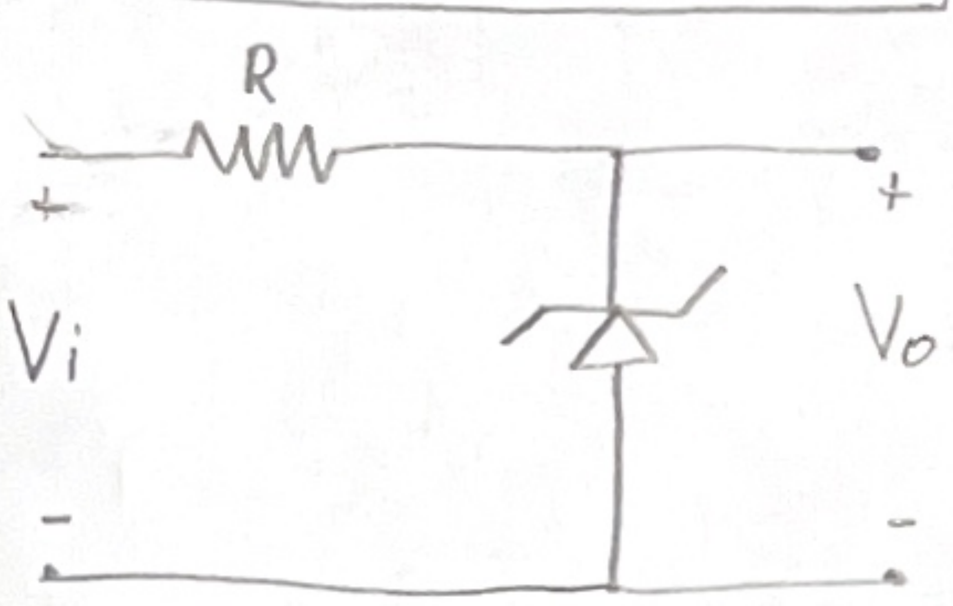
$$I = \frac{V_i - V_D}{R + r_D}$$

$$V_o = R\left(\frac{V_i - V_D}{R + r_D}\right)$$

Full Wave Bridge Rectifier



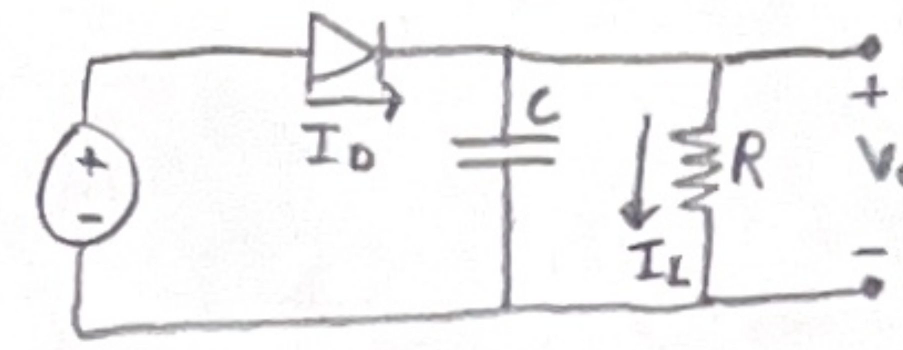
Zener diodes as Shunt Voltage regulators



$$\text{Line regulation} = \frac{\Delta V_o}{\Delta V_{in}}$$

$$\text{Load regulation} = \left|\frac{\Delta V_o}{\Delta I_L}\right|$$

Rectifier Circuit with resistive load



For Half Wave:

$$V_r = \frac{V_p T}{RC} = \frac{V_p}{fRC} = \frac{I_L}{fC}$$

For Full wave:

$$V_r = \frac{V_p T}{2RC} = \frac{V_p}{2fRC} = \frac{I_L}{2fC}$$

Load Voltage:

$$V_L = V_p - \frac{1}{2}V_r$$

Load Current:

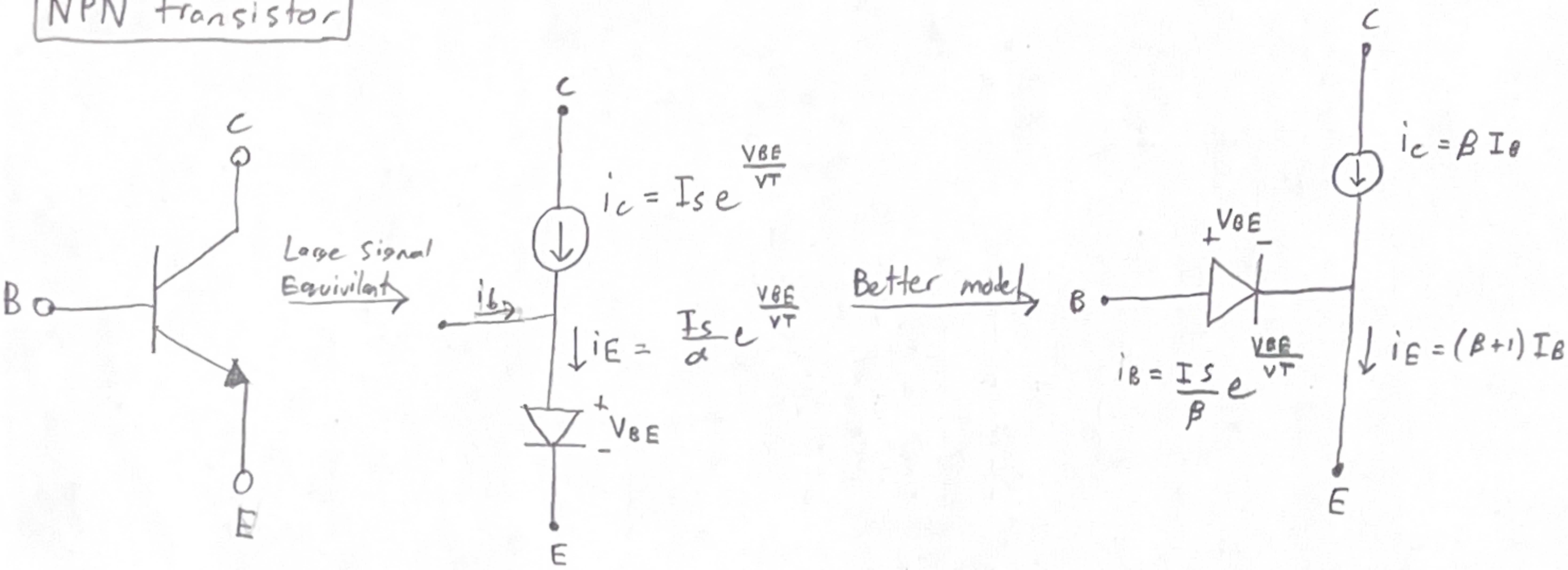
$$I_L = \frac{V_L}{R_L}$$

When ripple is small

$$I_L = \frac{V_p}{R_L}$$

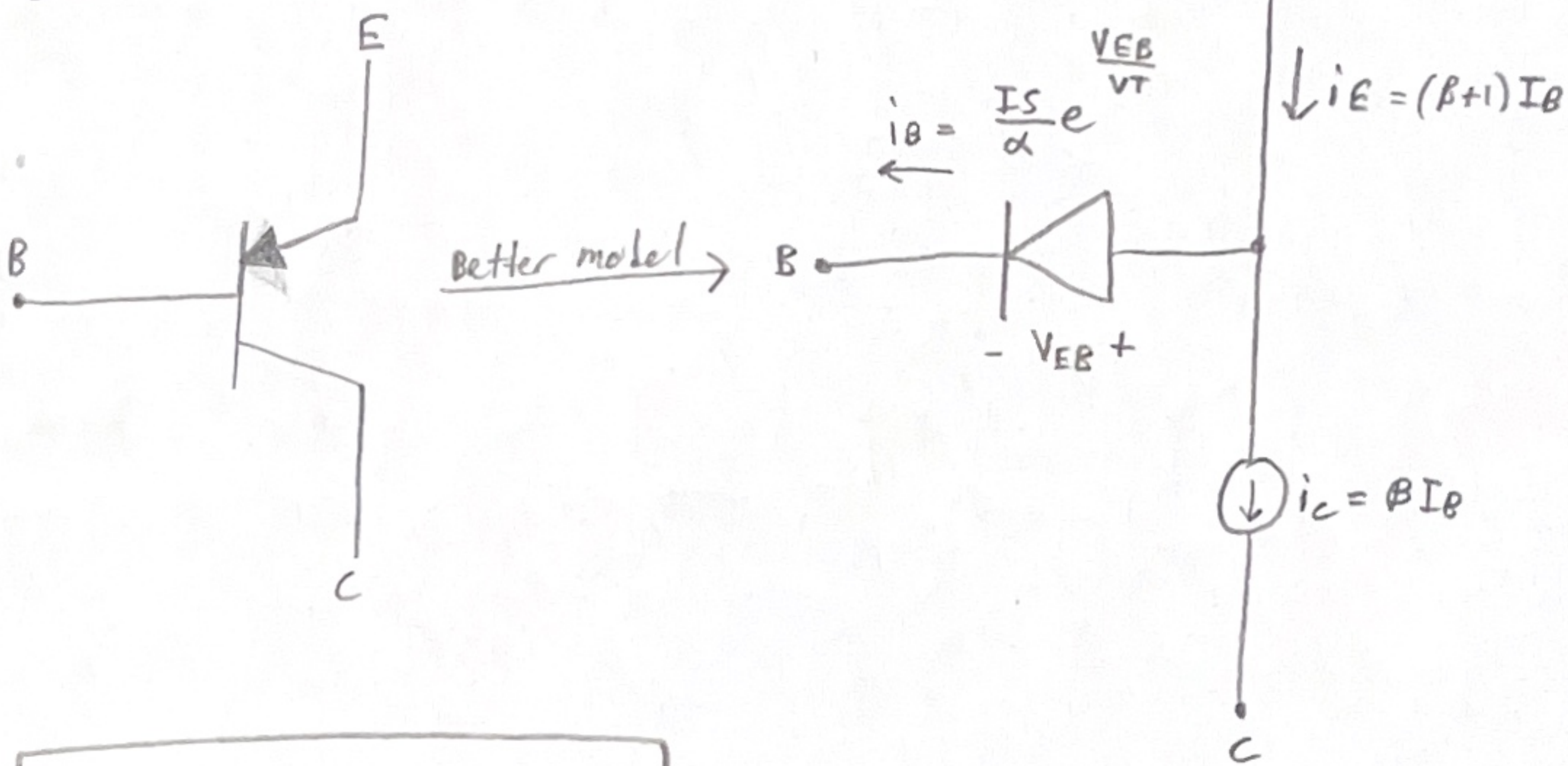
BJT's

NPN transistor



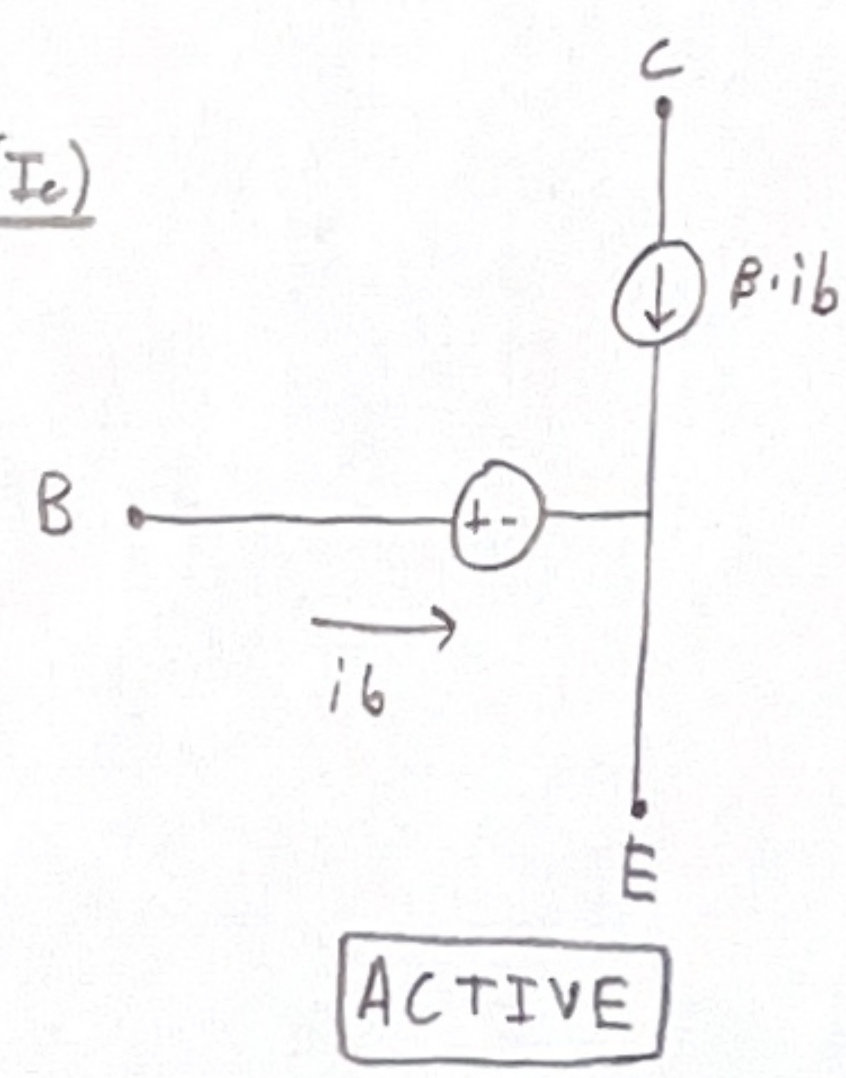
* only work in active region

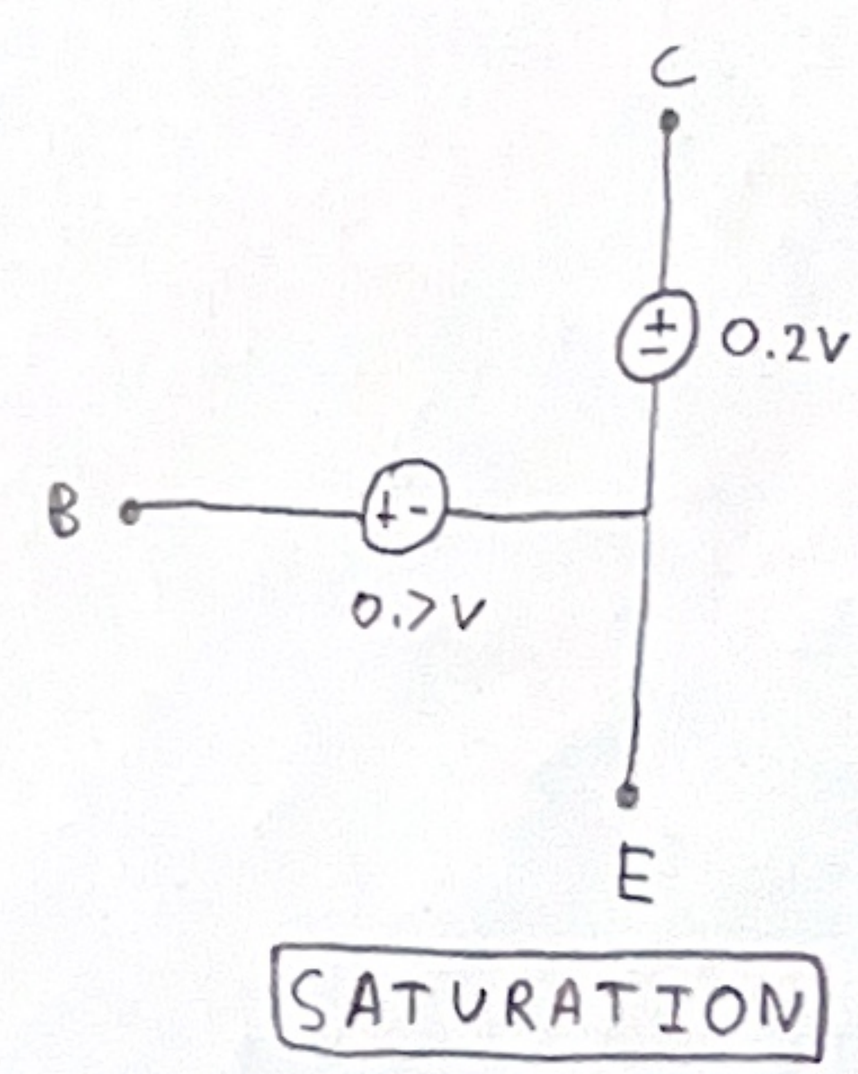
PNP Transistor



Modes of Operation

BJT Mode		Collector Current (I_c)
Cutoff	$\longrightarrow$	$i_c = 0$
Active	$\longrightarrow$	$i_c = \beta \cdot I_B$
Saturation	$\longrightarrow$	$i_c < \beta \cdot I_B$





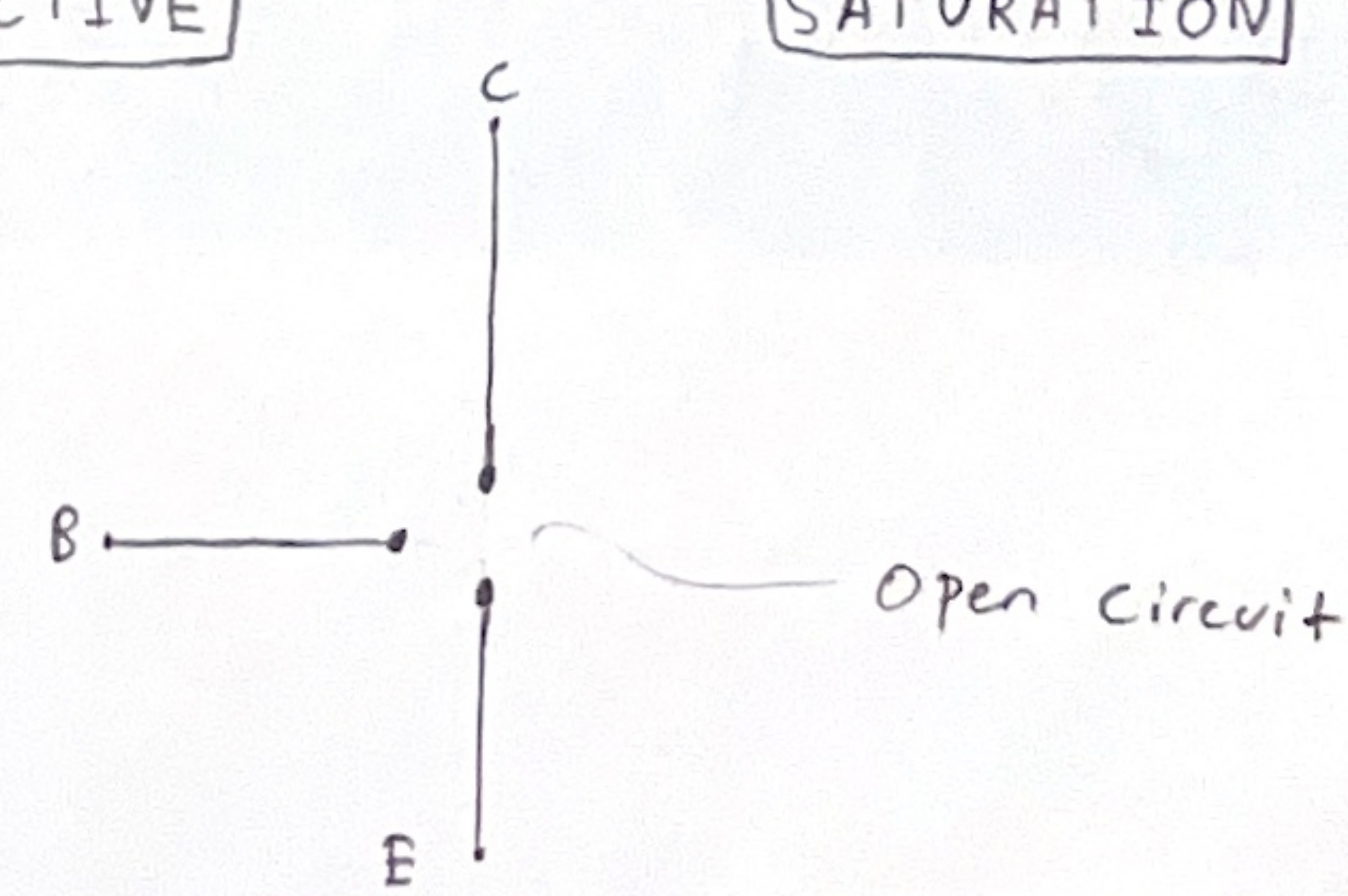
Relationships

$I_E = I_B + I_c$

$I_c = \beta \cdot I_B$

$I_c = \alpha \cdot I_E$

$\alpha = \frac{\beta}{\beta+1} \qquad \beta = \frac{\alpha}{1-\alpha}$



BJT Amplifiers

Steps:

① Get DC circuit:
- Short all AC Vsources
- Open all AC Isources
- open all capacitors
- short all inductors

② Solve circuit for I_C, r_π (or r_e), g_m:

③ Get AC circuit:
- replace transistor with hydrid-π circuit
- Short all DC Vsources
- Open all DC Isources
- Short all capacitors
- open all inductors

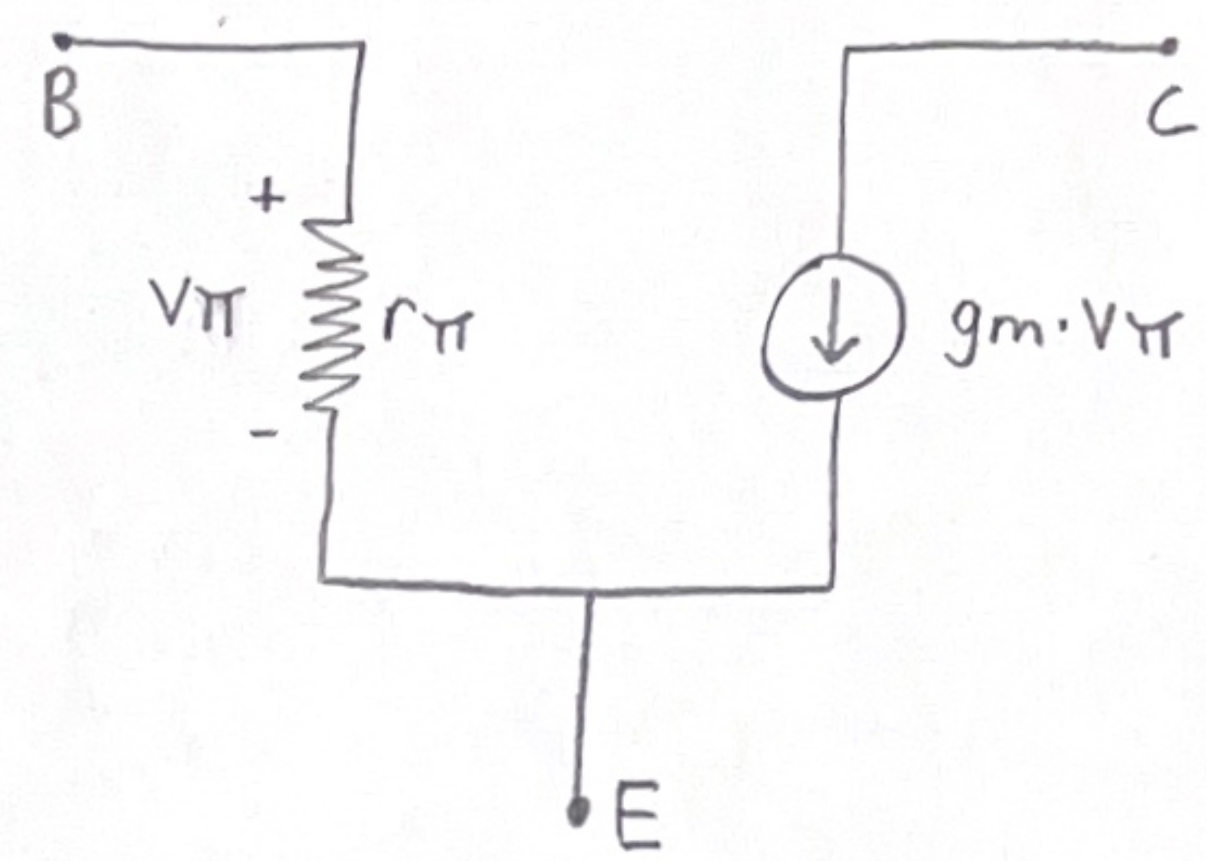
④ Solve AC circuit for (gain)

Models

$$g_m = \frac{I_C}{V_T}$$

$$r_\pi = \frac{\beta}{g_m}$$

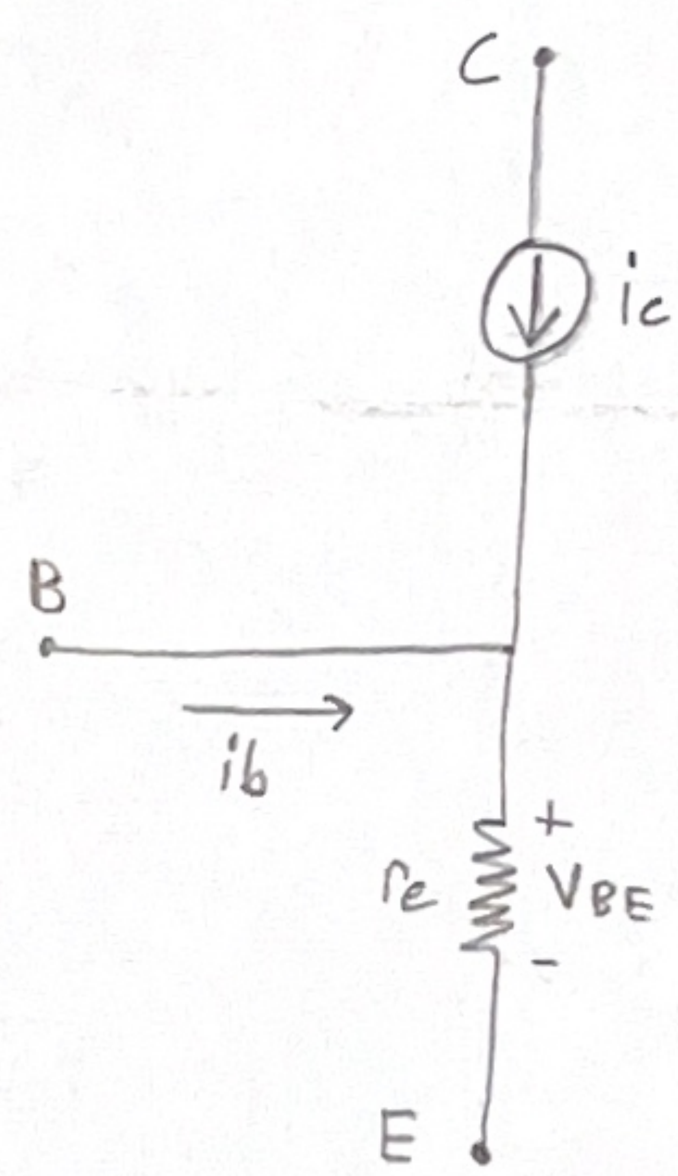
Hybrid-π model:



Input Resistance (R_i) → use hydrid-π model if R_i is measured from the base. (use r_π)

T-model:

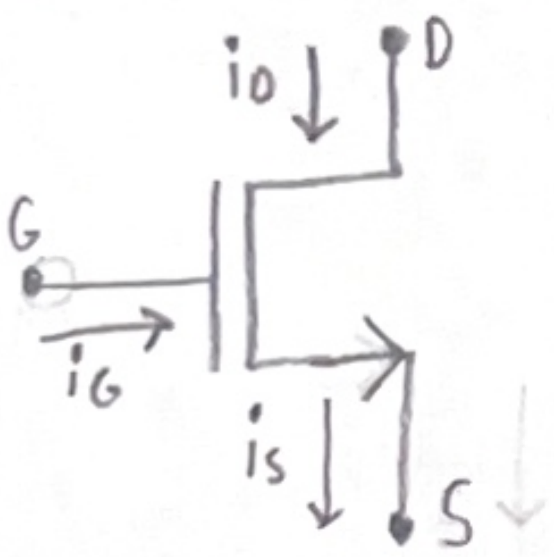
$$r_e = \frac{\alpha}{g_m}$$



Input Resistance (R_i) → use T-model if R_i is measured from the base. (use r_e)

MOSFETs

Enhancement N-MOSFET



Triode Region $V_{DS} < V_{GS} - V_t$

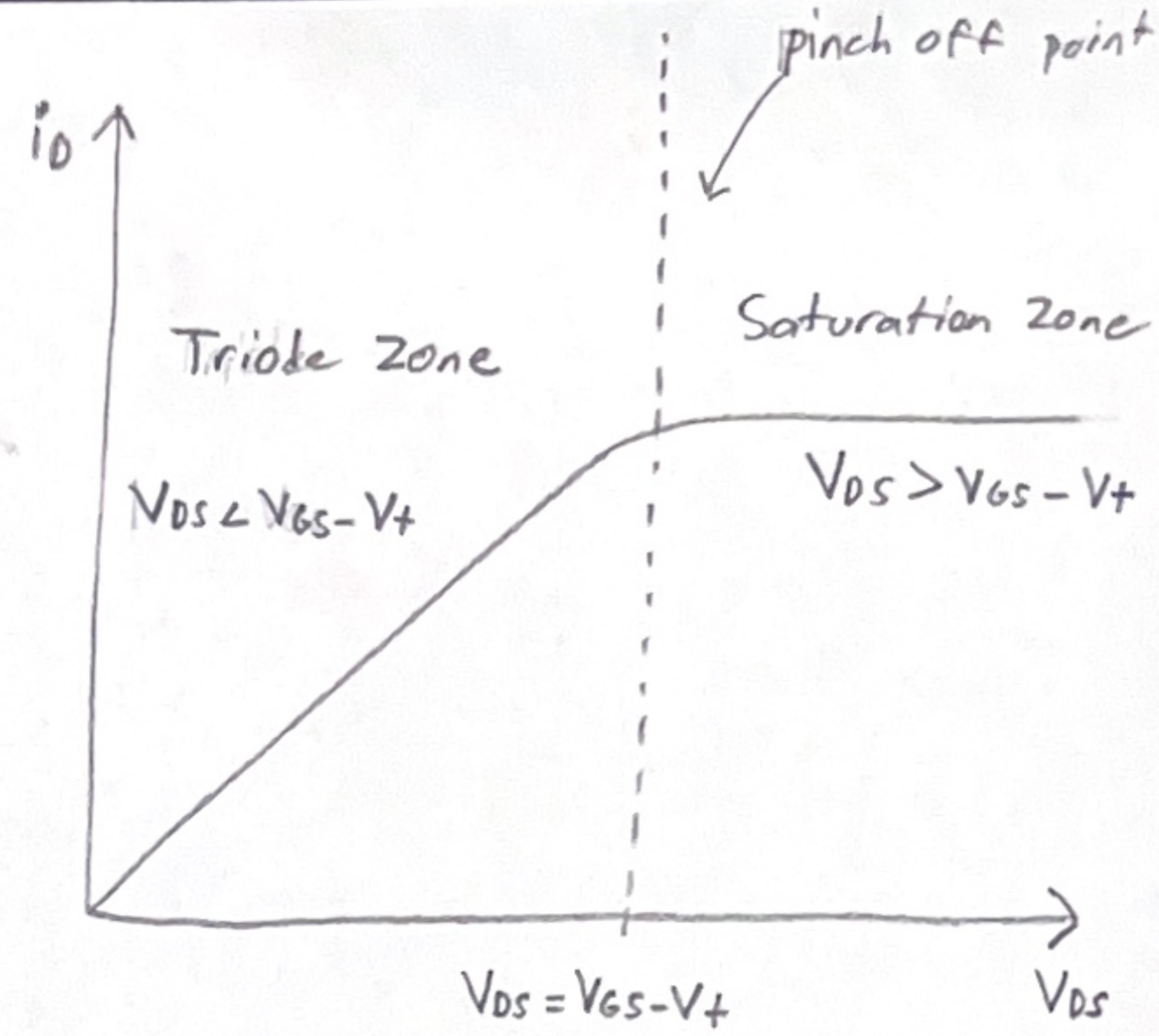
$$i_D = i_S = k_n' \frac{W}{L}\left[(V_{GS} - V_t)V_{DS} - \frac{1}{2}V_{DS}^2\right]$$

Saturation Region $V_{DS} > V_{GS} - V_t$

$$i_D = i_S = \frac{1}{2} k_n' \frac{W}{L}(V_{GS} - V_t)^2$$

Cutoff Region $V_{GS} < V_t$

$$i_D = 0$$



Small V_{DS} Assumption $V_{DS} < 0.2V$

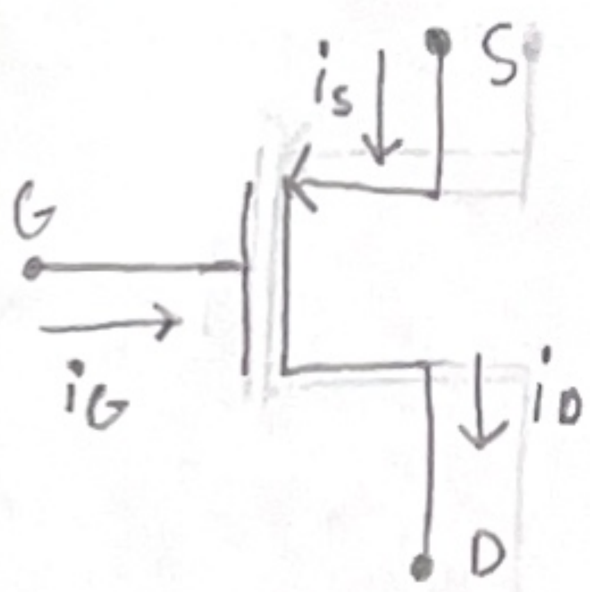
*in Triode Region

$$i_D \approx k_n' \frac{W}{L}\left[(V_{GS} - V_t)V_{DS}\right]$$

Where: $i_D \approx \frac{1}{r_{DS}} \cdot V_{DS}$

$$r_{ds} = \frac{1}{\mu_n C_{ox} \frac{W}{L}(V_{GS} - V_t)}$$

Enhancement P-MOSFET



Triode Region $|V_{DS}| < |V_{GS} - V_t|$

$$i_D = i_S = k_p' \frac{W}{L}\left[(|V_{GS} - V_t|)|V_{DS}| - \frac{1}{2}V_{DS}^2\right]$$

Saturation Region $|V_{DS}| > |V_{GS} - V_t|$

$$i_D = i_S = \frac{1}{2} k_n' \frac{W}{L}(V_{GS} - V_t)^2$$

- With Early Voltage:

$\lambda = \frac{1}{V_A}$ where $V_A \neq \infty$

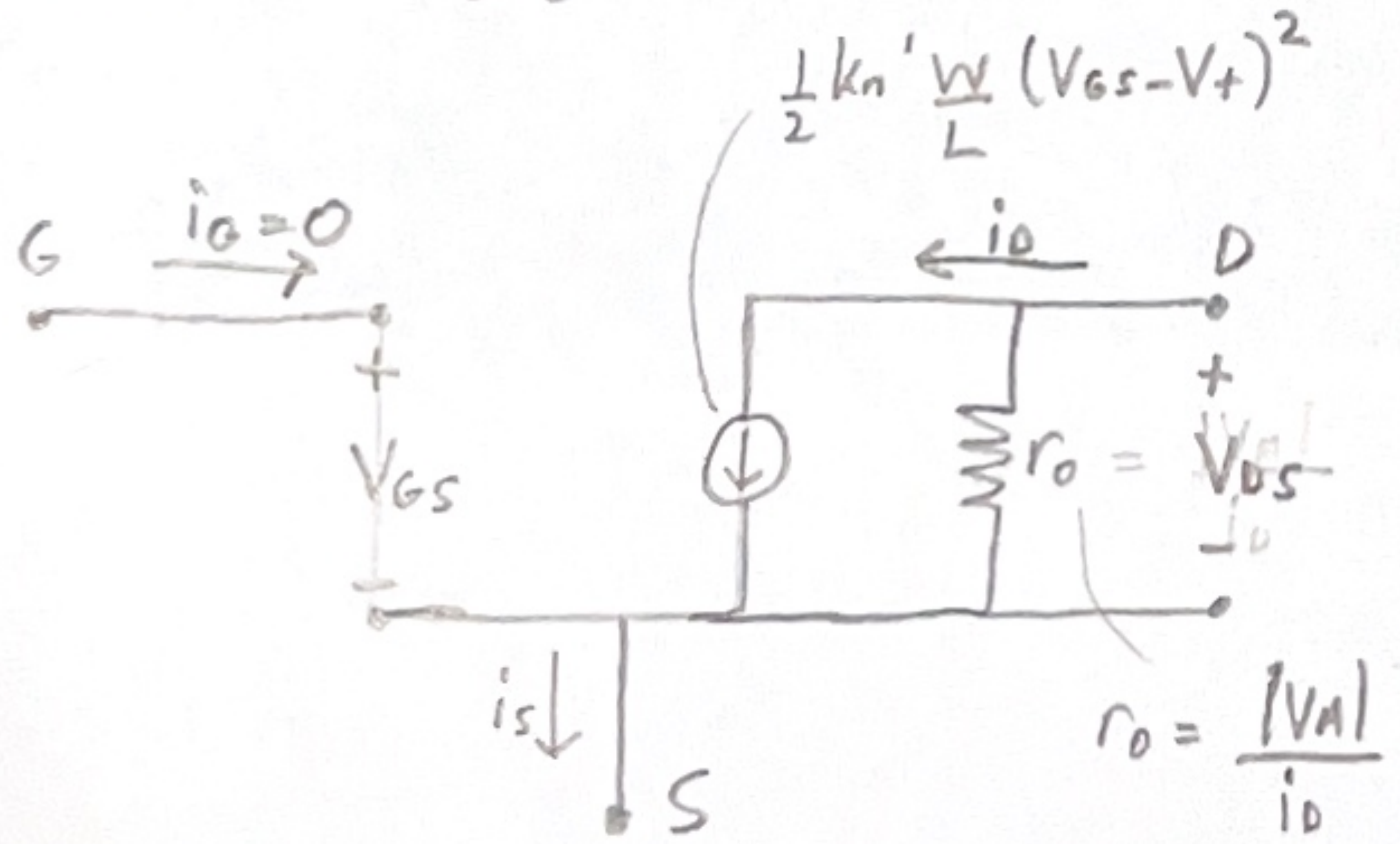
$$i_D = i_S = \frac{1}{2} k_n' \frac{W}{L}(V_{GS} - V_t)^2(1 + |\lambda||V_{DS}|)$$

Cutoff Region $|V_{GS}| < |V_t|$

$$i_D = i_G = i_S = 0$$

Large Signal Model

* Enhancement N-MOSFET in Saturation mode

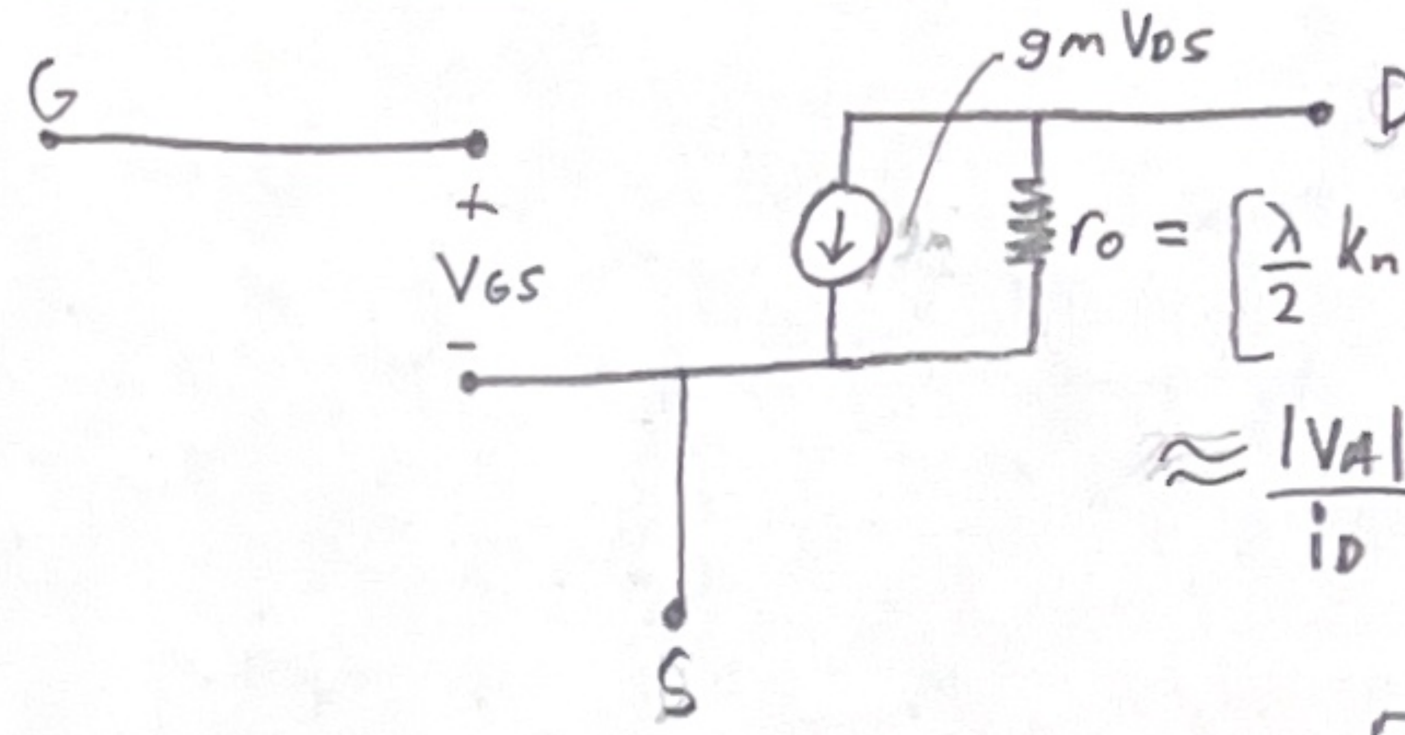


AC and DC Analysis

Steps:

1. Replace with DC Circuit
2. Solve DC Circuit for V_{GS}, i_D, V_{DS}. Make sure the MOSFET is in saturation
3. Replace with AC small signal model
4. Solve for Voltage gain/current gain, etc

AC Small signal model



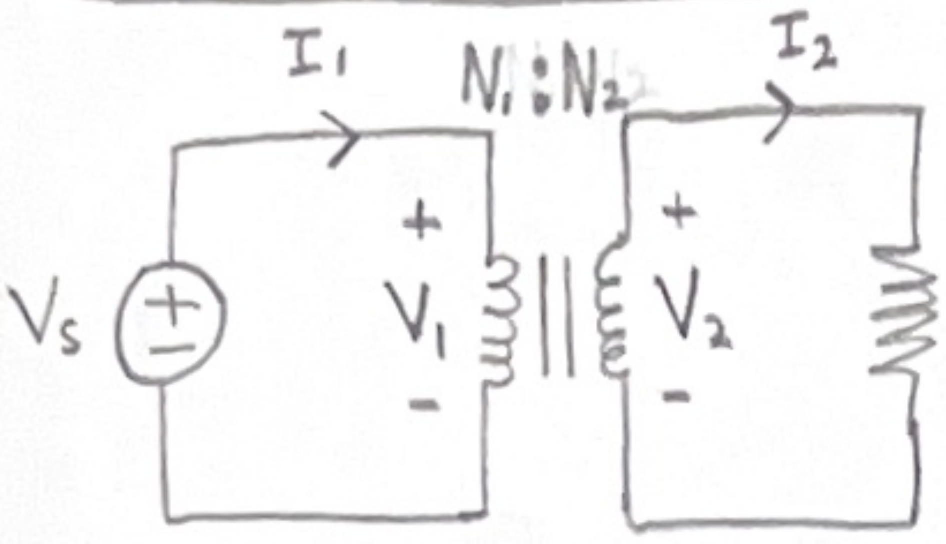
$$r_o = \left[\frac{\lambda}{2} k_n' \frac{W}{L} (V_{GS}-V_t)^2\right]^{-1} \approx \frac{|V_A|}{i_D}$$

Formulas

$$\lambda = \frac{1}{V_A}$$

$$g_m = k_n' \frac{W}{L}(V_{GS}-V_t)$$

Ideal Transformers



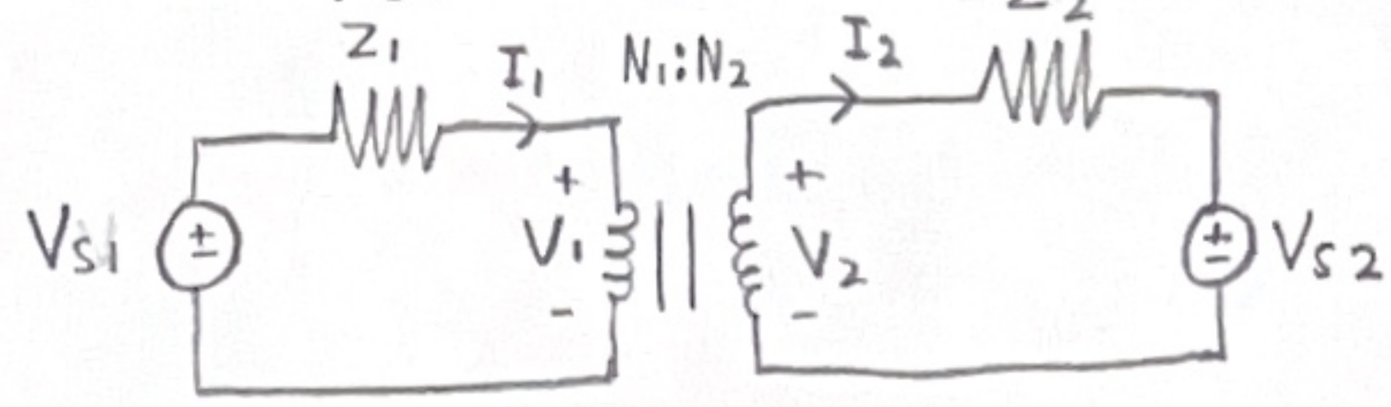
*Note :

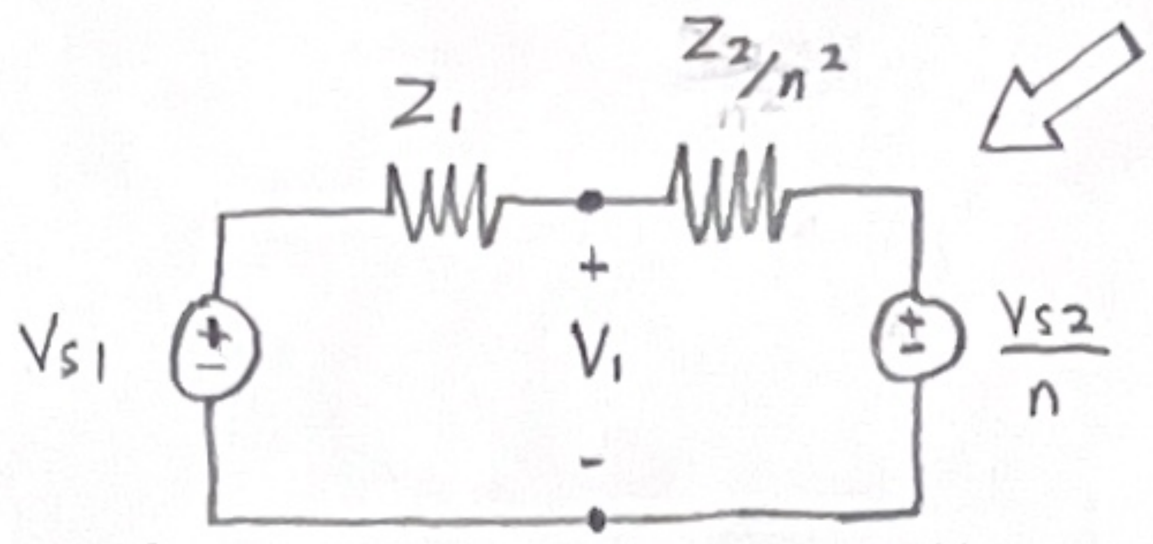
$N_1 : N_2$

$\downarrow$

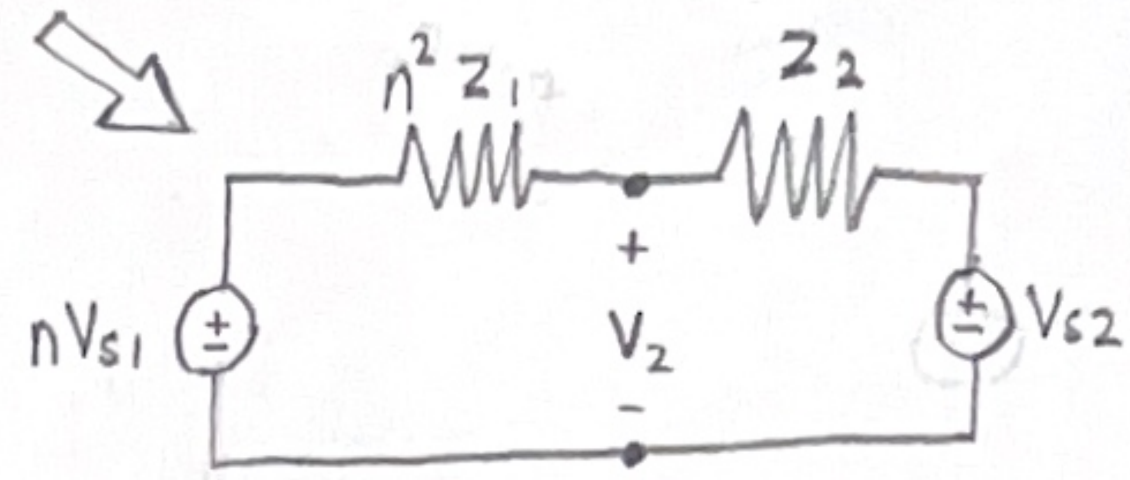
$1 : n$

$n = \frac{N_2}{N_1}$





"View from Source"



"View from Load"

① $\frac{V_2}{V_1} = \frac{N_2}{N_1}$ ② $\frac{I_2}{I_1} = \frac{N_1}{N_2}$

Laplace Tables

Property	$f(t)$	$F(s)$
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(s) + a_2 F_2(s)$
Scaling	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
Time shift	$f(t-a)u(t-a)$	$e^{-as}F(s)$
Frequency shift	$e^{-at}f(t)$	$F(s+a)$
Time differentiation	$\frac{df}{dt}$	$sF(s) - f(0^-)$
	$\frac{d^2 f}{dt^2}$	$s^2F(s) - sf(0^-) - f'(0^-)$
	$\frac{d^3 f}{dt^3}$	$s^3F(s) - s^2 f(0^-) - sf'(0^-) - f''(0^-)$
	$\frac{d^n f}{dt^n}$	$s^nF(s) - s^{n-1}f(0^-) - s^{n-2}f'(0^-) - \cdots - f^{(n-1)}(0^-)$
Time integration	$\int_0^t f(t)dt$	$\frac{1}{s}F(s)$
Frequency differentiation	$tf(t)$	$-\frac{d}{ds}F(s)$
Frequency integration	$\frac{f(t)}{t}$	$\int_s^\infty F(s)ds$
Time periodicity	$f(t) = f(t+nT)$	$\frac{F_1(s)}{1-e^{-sT}}$
Initial value	$f(0)$	$\lim_{s\to\infty} sF(s)$
Final value	$f(\infty)$	$\lim_{s\to 0} sF(s)$
Convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$

Laplace transform pairs.*

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
t	$\frac{1}{s^2}$
t^n	$\frac{n!}{s^{n+1}}$
te^{-at}	$\frac{1}{(s+a)^2}$
$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
$\sin(\omega t + \theta)$	$\frac{s \sin\theta + \omega \cos\theta}{s^2+\omega^2}$
$\cos(\omega t + \theta)$	$\frac{s \cos\theta - \omega \sin\theta}{s^2+\omega^2}$
$e^{-at}\sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos \omega t$	$\frac{s+a}{(s+a)^2+\omega^2}$

*Defined for $t \geq 0$; $f(t) = 0$, for $t < 0$.

2nd Order Circuits

$s^2 + 2\alpha s + \omega_0^2 \longrightarrow$

$\alpha > \omega_0$ OVERDAMPED

$\alpha = \omega_0$ CRITICALLY DAMPED

$\alpha < \omega_0$ UNDERDAMPED

① $i_L(0^+)$ and $V_C(0^+)$ found for $t < 0$ (long time)

② $i_L'(0^+)$ and $V_C'(0^+)$ found for $t > 0$ (after switch)

RMS Values

$V_{rms} = \frac{1}{\sqrt{2}} V_p$ (Peak is bigger than rms)

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [Z] \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \qquad \begin{bmatrix} V_1 \\ i_2 \end{bmatrix} = [h] \begin{bmatrix} i_1 \\ V_2 \end{bmatrix} \qquad \begin{bmatrix} V_1 \\ i_1 \end{bmatrix} = [T] \begin{bmatrix} V_2 \\ -i_2 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = [y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \qquad \begin{bmatrix} i_1 \\ V_2 \end{bmatrix} = [g] \begin{bmatrix} V_1 \\ i_2 \end{bmatrix} \qquad \begin{bmatrix} V_2 \\ i_2 \end{bmatrix} = [t] \begin{bmatrix} V_1 \\ -i_1 \end{bmatrix}$$

Impedance Parameters;

Apply 1A to V_1 side	Apply 1A to V_2 side
$V_1 = Z_{11}$	$V_2 = Z_{22}$
$V_2 = Z_{21}$	$V_1 = Z_{12}$

TABLE 19.1

Conversion of two-port parameters.

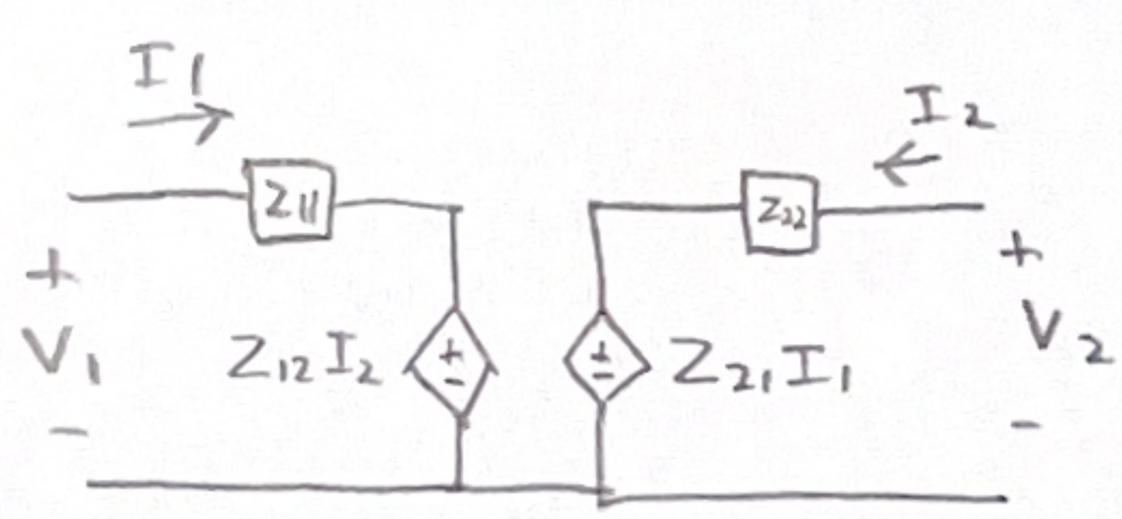
	Impedance **z**		Admittance **y**		Hybrid **h**		Inverse Hybrid **g**		Transmission **T**		Inverse Transmission **t**	
z	z_{11}	z_{12}	$\frac{y_{22}}{\Delta_y}$	$-\frac{y_{12}}{\Delta_y}$	$\frac{\Delta_h}{h_{22}}$	$\frac{h_{12}}{h_{22}}$	$\frac{1}{g_{11}}$	$-\frac{g_{12}}{g_{11}}$	$\frac{A}{C}$	$\frac{\Delta_T}{C}$	$\frac{d}{c}$	$\frac{1}{c}$
	z_{21}	z_{22}	$-\frac{y_{21}}{\Delta_y}$	$\frac{y_{11}}{\Delta_y}$	$-\frac{h_{21}}{h_{22}}$	$\frac{1}{h_{22}}$	$\frac{g_{21}}{g_{11}}$	$\frac{\Delta_g}{g_{11}}$	$\frac{1}{C}$	$\frac{D}{C}$	$\frac{\Delta_t}{c}$	$\frac{a}{c}$
y	$\frac{z_{22}}{\Delta_z}$	$-\frac{z_{12}}{\Delta_z}$	y_{11}	y_{12}	$\frac{1}{h_{11}}$	$-\frac{h_{12}}{h_{11}}$	$\frac{\Delta_g}{g_{22}}$	$\frac{g_{12}}{g_{22}}$	$\frac{D}{B}$	$-\frac{\Delta_T}{B}$	$\frac{a}{b}$	$-\frac{1}{b}$
	$-\frac{z_{21}}{\Delta_z}$	$\frac{z_{11}}{\Delta_z}$	y_{21}	y_{22}	$\frac{h_{21}}{h_{11}}$	$\frac{\Delta_h}{h_{11}}$	$-\frac{g_{21}}{g_{22}}$	$\frac{1}{g_{22}}$	$-\frac{1}{B}$	$\frac{A}{B}$	$-\frac{\Delta_t}{b}$	$\frac{d}{b}$
h	$\frac{\Delta_z}{z_{22}}$	$\frac{z_{12}}{z_{22}}$	$\frac{1}{y_{11}}$	$-\frac{y_{12}}{y_{11}}$	h_{11}	h_{12}	$\frac{g_{22}}{\Delta_g}$	$-\frac{g_{12}}{\Delta_g}$	$\frac{B}{D}$	$\frac{\Delta_T}{D}$	$\frac{b}{a}$	$\frac{1}{a}$
	$-\frac{z_{21}}{z_{22}}$	$\frac{1}{z_{22}}$	$\frac{y_{21}}{y_{11}}$	$\frac{\Delta_y}{y_{11}}$	h_{21}	h_{22}	$-\frac{g_{21}}{\Delta_g}$	$\frac{g_{11}}{\Delta_g}$	$-\frac{1}{D}$	$\frac{C}{D}$	$\frac{\Delta_t}{a}$	$\frac{c}{a}$
g	$\frac{1}{z_{11}}$	$-\frac{z_{12}}{z_{11}}$	$\frac{\Delta_y}{y_{22}}$	$\frac{y_{12}}{y_{22}}$	$\frac{h_{22}}{\Delta_h}$	$-\frac{h_{12}}{\Delta_h}$	g_{11}	g_{12}	$\frac{C}{A}$	$-\frac{\Delta_T}{A}$	$\frac{c}{d}$	$-\frac{1}{d}$
	$\frac{z_{21}}{z_{11}}$	$\frac{\Delta_z}{z_{11}}$	$-\frac{y_{21}}{y_{22}}$	$\frac{1}{y_{22}}$	$-\frac{h_{21}}{\Delta_h}$	$\frac{h_{11}}{\Delta_h}$	g_{21}	g_{22}	$\frac{1}{A}$	$\frac{B}{A}$	$\frac{\Delta_t}{d}$	$-\frac{b}{d}$
T	$\frac{z_{11}}{z_{21}}$	$\frac{\Delta_z}{z_{21}}$	$-\frac{y_{22}}{y_{21}}$	$-\frac{1}{y_{21}}$	$-\frac{\Delta_h}{h_{21}}$	$-\frac{h_{11}}{h_{21}}$	$\frac{1}{g_{21}}$	$\frac{g_{22}}{g_{21}}$	A	B	$\frac{d}{\Delta_t}$	$\frac{b}{\Delta_t}$
	$\frac{1}{z_{21}}$	$\frac{z_{22}}{z_{21}}$	$-\frac{\Delta_y}{y_{21}}$	$-\frac{y_{11}}{y_{21}}$	$-\frac{h_{22}}{h_{21}}$	$-\frac{1}{h_{21}}$	$\frac{g_{11}}{g_{21}}$	$\frac{\Delta_g}{g_{21}}$	C	D	$\frac{c}{\Delta_t}$	$\frac{a}{\Delta_t}$
t	$\frac{z_{22}}{z_{12}}$	$\frac{\Delta_z}{z_{12}}$	$-\frac{y_{11}}{y_{12}}$	$-\frac{1}{y_{12}}$	$\frac{1}{h_{12}}$	$\frac{h_{11}}{h_{12}}$	$-\frac{\Delta_g}{g_{12}}$	$-\frac{g_{22}}{g_{12}}$	$\frac{D}{\Delta_T}$	$\frac{B}{\Delta_T}$	a	b
	$\frac{1}{z_{12}}$	$\frac{z_{11}}{z_{12}}$	$-\frac{\Delta_y}{y_{12}}$	$-\frac{y_{22}}{y_{12}}$	$\frac{h_{22}}{h_{12}}$	$\frac{\Delta_h}{h_{12}}$	$-\frac{g_{11}}{g_{12}}$	$-\frac{1}{g_{12}}$	$\frac{C}{\Delta_T}$	$\frac{A}{\Delta_T}$	c	d

$\Delta_z = z_{11}z_{22} - z_{12}z_{21}$, $\Delta_h = h_{11}h_{22} - h_{12}h_{21}$, $\Delta_T = AD - BC$
$\Delta_y = y_{11}y_{22} - y_{12}y_{21}$, $\Delta_g = g_{11}g_{22} - g_{12}g_{21}$, $\Delta_t = ad - bc$

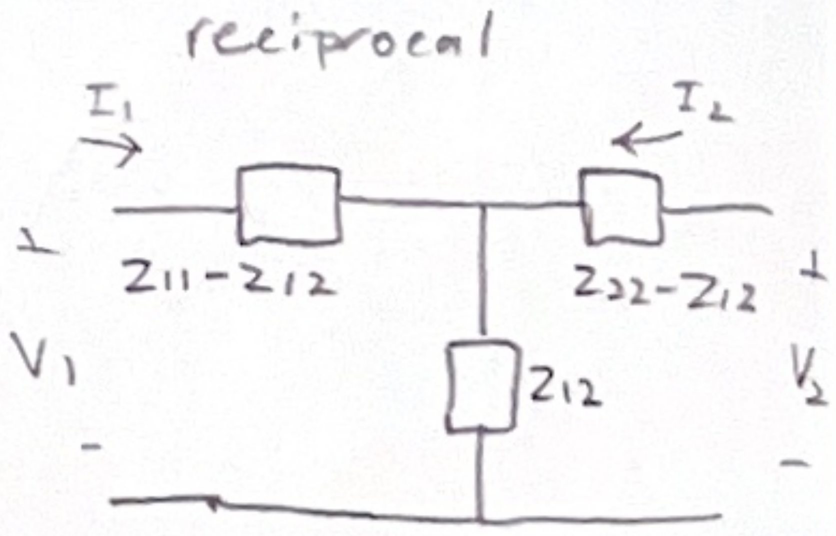
$\boxed{\text{Combining networks}}$

1) Series, impedances add
2) Parallel, Admittances add
3) Cascade, Transmissions multiply

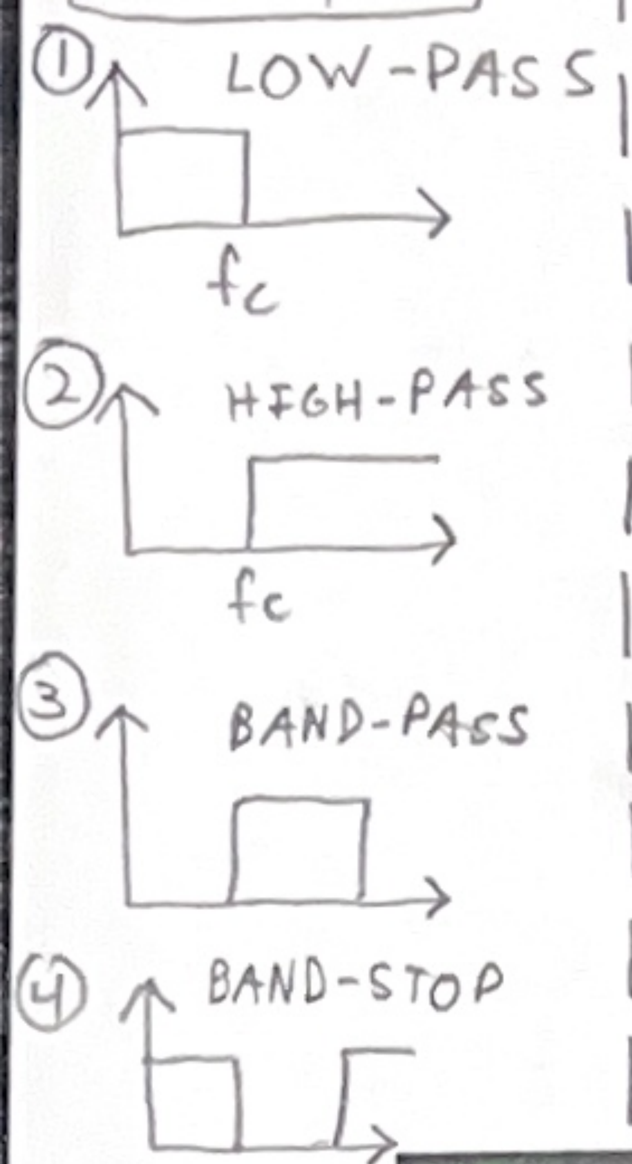
$\boxed{\text{Model}}$



*if $Z_{12} = Z_{21}$ reciprocal



4 Types

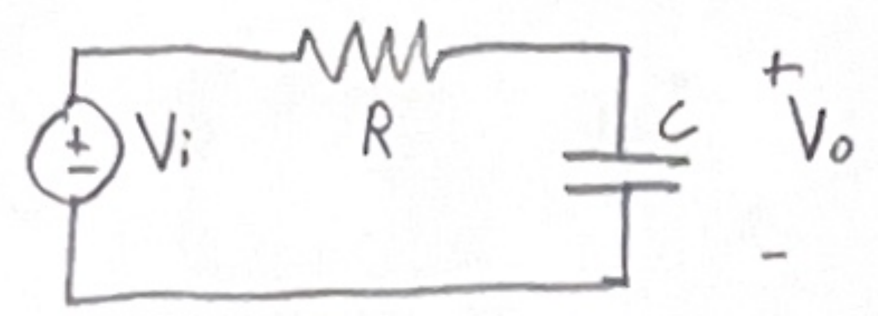


Filter

- Active (OP-Amp, Transistor)
- Passive (Resistor, capacitor, inductor)

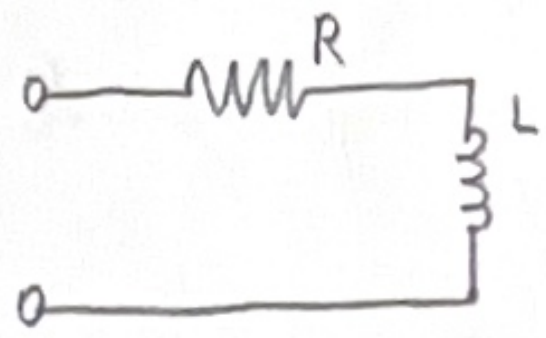
LOW-PASS Filter

RC LOW-Pass :



RL Low Pass

RL High Pass



RC High Pass

RLC Filter

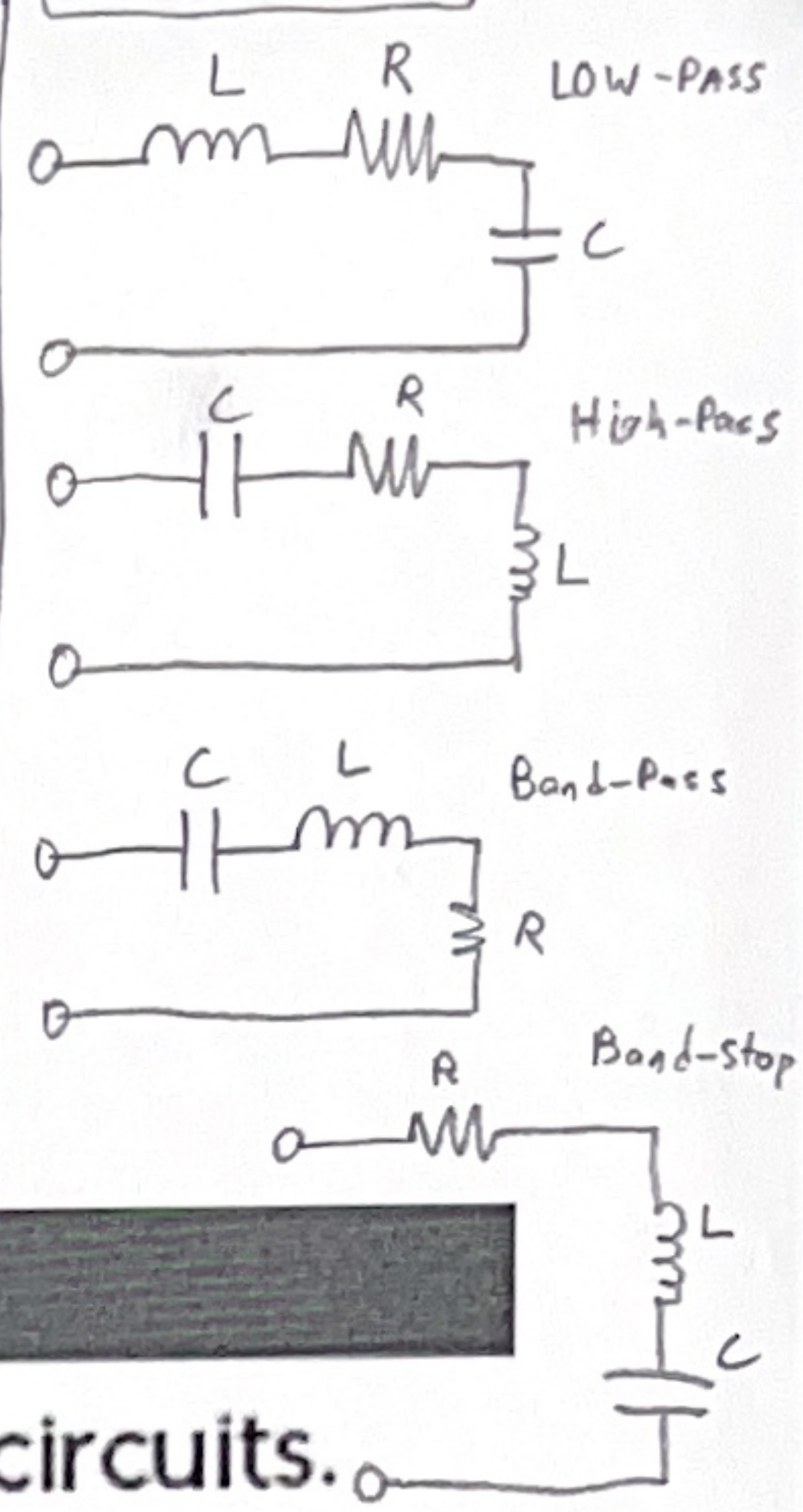


TABLE 14.4

Summary of the characteristics of resonant *RLC* circuits.

Characteristic	Series circuit	Parallel circuit
Resonant frequency, ω_0	$\dfrac{1}{\sqrt{LC}}$	$\dfrac{1}{\sqrt{LC}}$
(Flipped) Quality factor, Q	$\dfrac{\omega_0 L}{R}$ or $\dfrac{1}{\omega_0 RC}$	$\dfrac{R}{\omega_0 L}$ or $\omega_0 RC$
Bandwidth, B — W_2, W_1 = Poles of $H(s)$; $W_0 = \sqrt{W_1 \cdot W_2}$; $BW = W_2 - W_1$	$\dfrac{\omega_0}{Q}$	$\dfrac{\omega_0}{Q}$
Half-power frequencies, ω_1, ω_2 — $BW = W_2 - W_1$	$\omega_0\sqrt{1+\left(\dfrac{1}{2Q}\right)^2} \pm \dfrac{\omega_0}{2Q}$	$\omega_0\sqrt{1+\left(\dfrac{1}{2Q}\right)^2} \pm \dfrac{\omega_0}{2Q}$
For $Q \geq 10$, ω_1, ω_2	$\omega_0 \pm \dfrac{B}{2}$	$\omega_0 \pm \dfrac{B}{2}$

Scaling

$K_f = \dfrac{f_{new}}{f_{old}}$

$L' = \dfrac{L_{old}}{K_f}$

$C' = \dfrac{K_f}{C_{old}}$

(changed frequency)

(Want to scale up components)

$R' = K_m \cdot R$

$L' = K_m \cdot L$

$C' = \dfrac{C}{K_m}$

(change both)(General)

$R' = K_m \cdot R$

$L' = \dfrac{K_m}{K_f} \cdot L$

$C' = \dfrac{1}{K_m \cdot K_f} \cdot C$